

The equation of state of QCD at finite temperature and chemical potential(s)

Michael Strickland

Primary References and Collaborators

1309.3968: N. Haque, J.O. Andersen, M.G. Mustafa, MS, and N. Su (3-loop HTLpt – Short Version)

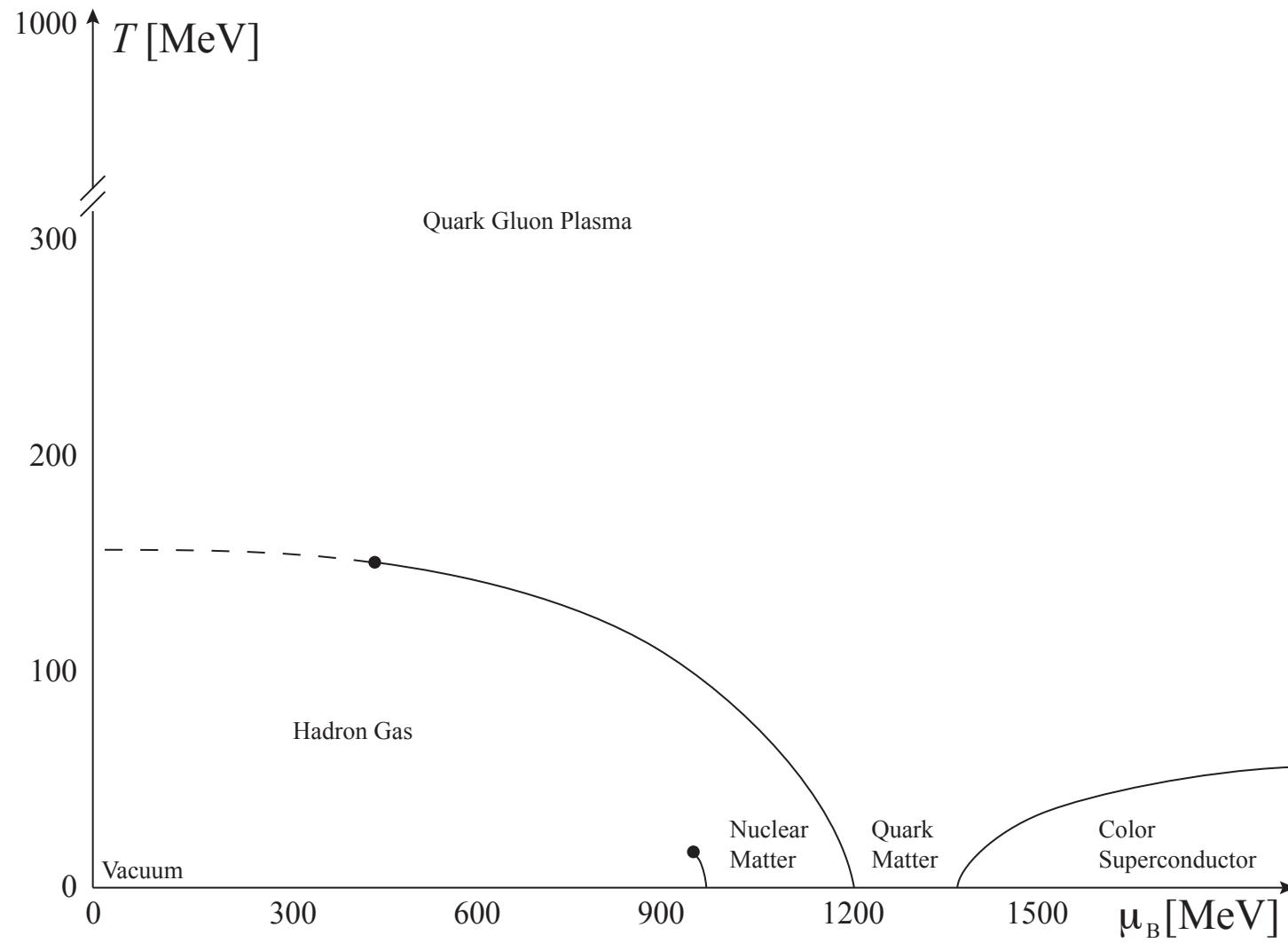
1307.8098: S. Mogliacci, J.O. Andersen, N. Su, MS, and A. Vuorinen (4-loop resummed DR)

1402.6907: N. Haque, A. Bandyopadhyay, J.O. Andersen, M.G. Mustafa, MS, N. Su (3-loop HTLpt – Long Version)

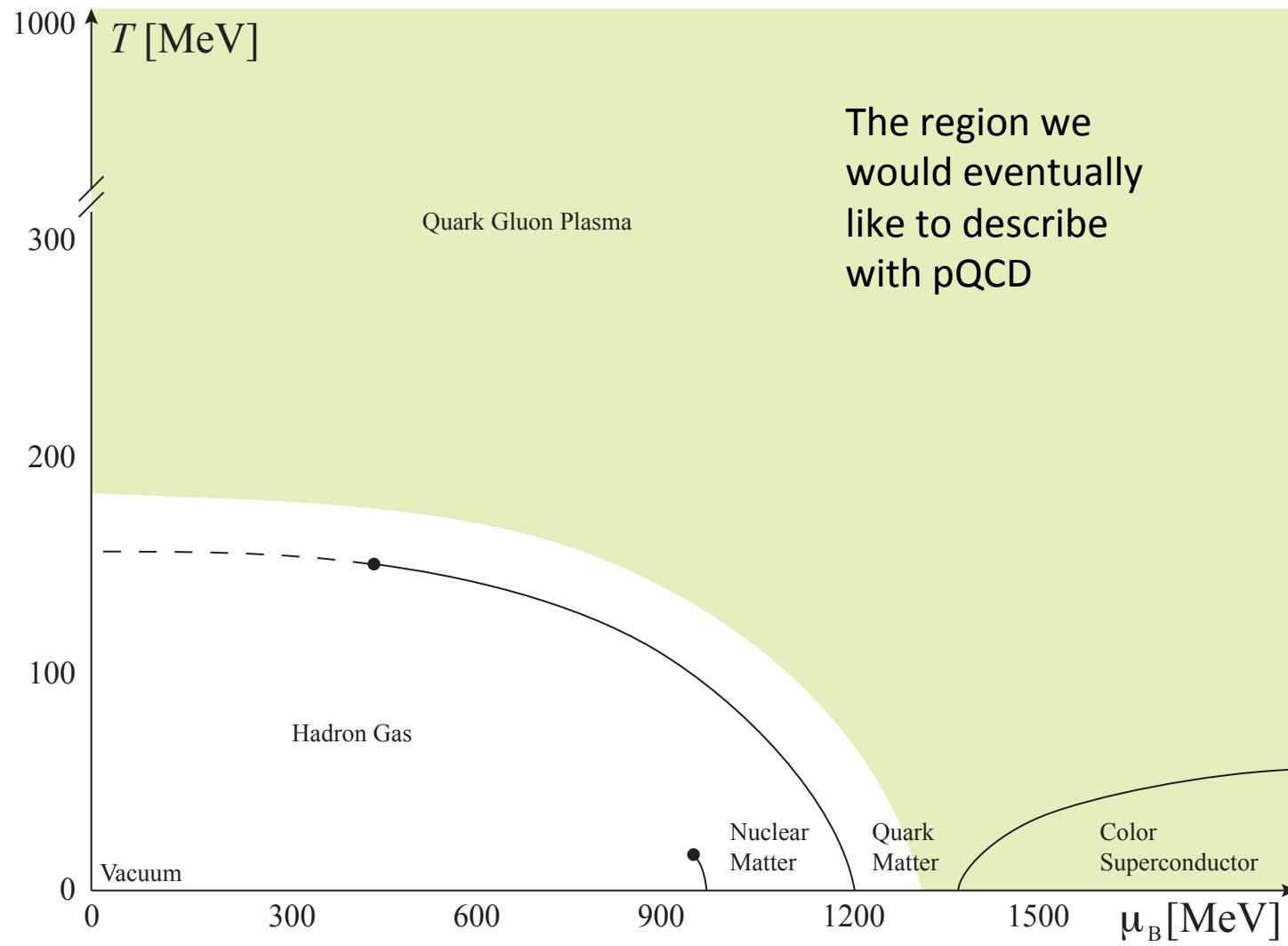
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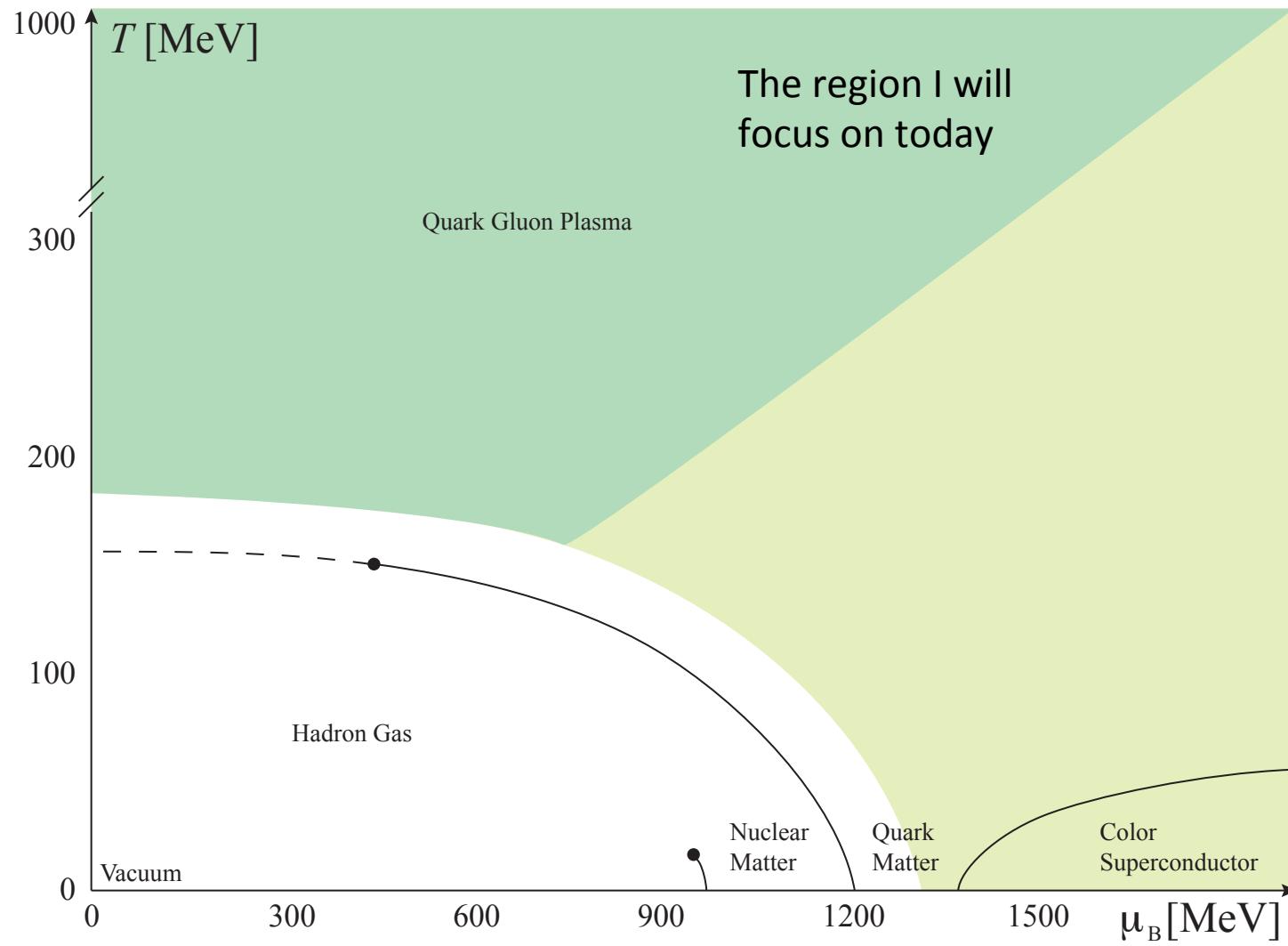
QCD Phase Diagram



QCD Phase Diagram

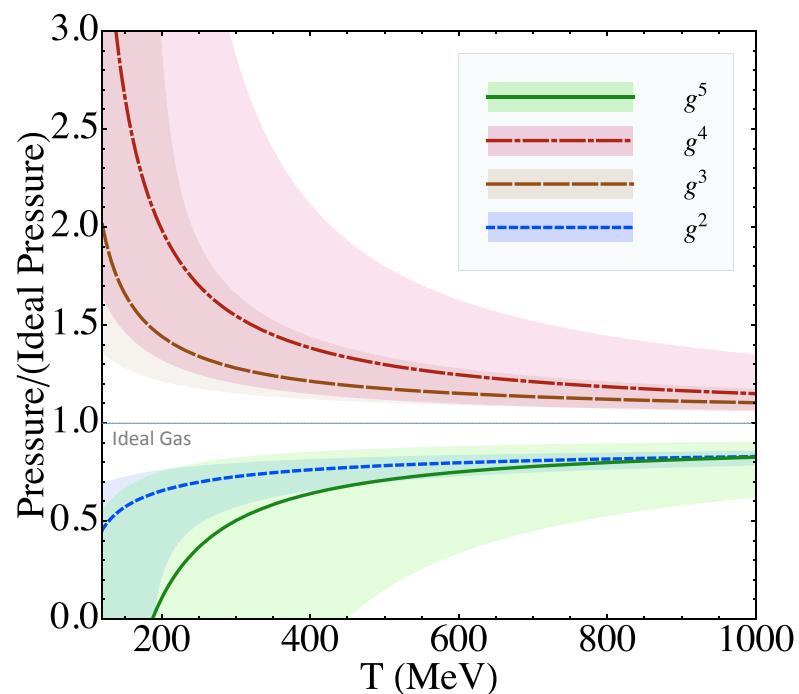


QCD Phase Diagram



Naïve pQCD thermodynamics

- QCD free energy at finite T known up to three loops (g^5) since 1994
(Shuryak & Chin, Kapusta, Toimela, Arnold, Zhai, Khaestening, Braaten, Nieto)
- Coefficient at $O(g^6 \log g)$ also known
(Kajantie, Laine, Rummukainen, Schroder)
- Extension to finite chemical potential
(Vuorinen)
- Very poorly convergent: need temperatures on the order of $T \sim 10^5$ GeV for convergence
- Similar problem in QED and scalar theories → the problem is not specific to QCD
- Our goal: Find a more convergent gauge-invariant scheme for $T > 2T_c$



- Resulting framework should also be able to describe dynamical properties of the QGP

Simple Case – Anharmonic Oscillator

- Consider quantum mechanics in an anharmonic potential

$$V(x) = \frac{1}{2}\omega^2 x^2 + \frac{g}{4}x^4 \quad (\omega^2, g > 0)$$

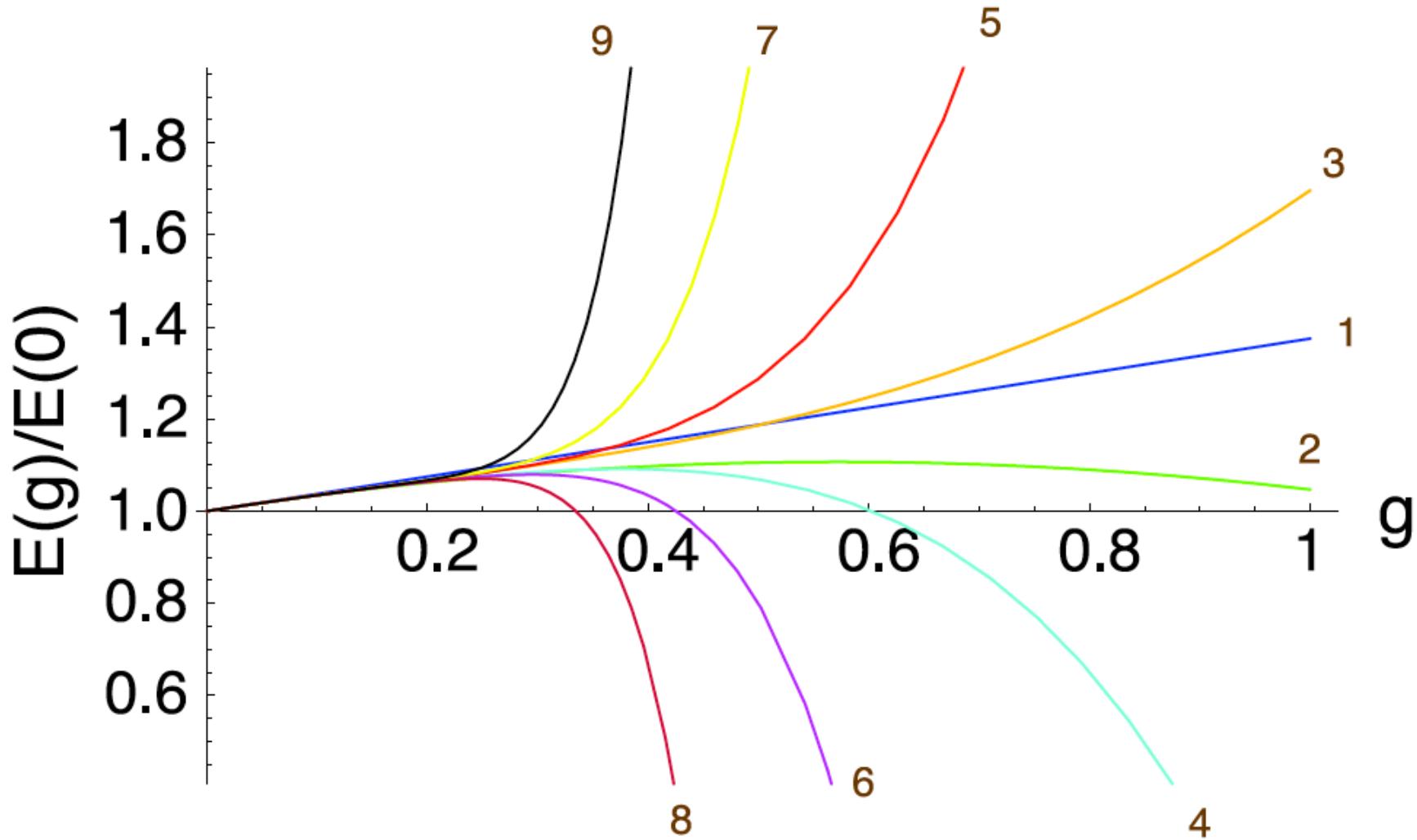
- Weak-coupling expansion of the ground state energy is known up to all orders (Bender and Wu 69/73)

$$E(g) = \omega \sum_{n=0}^{\infty} c_n^{\text{BW}} \left(\frac{g}{4\omega^3} \right)^n, \quad c_n^{\text{BW}} = \left\{ \frac{1}{2}, \frac{3}{4}, -\frac{21}{8}, \frac{333}{16}, -\frac{30885}{16}, \dots \right\}$$

$$\lim_{n \rightarrow \infty} c_n^{\text{BW}} = (-1)^{n+1} \sqrt{\frac{6}{\pi^3}} 3^n \left(n - \frac{1}{2}\right)!$$

- Factorial growth → expansion is an asymptotic series with zero radius of convergence!

Asymptotic Series



Variational Perturbation Theory

- Split the harmonic term into two pieces and treat the second as part of the interaction (Janke and Kleinert 95)

$$\omega^2 \rightarrow \Omega^2 + (\omega^2 - \Omega^2) \implies E_N(g, r) = \Omega \sum_{n=0}^N c_n(r) \left(\frac{g}{4\Omega^3} \right)^n$$
$$r \equiv \frac{2}{g} (\omega^2 - \Omega^2)$$

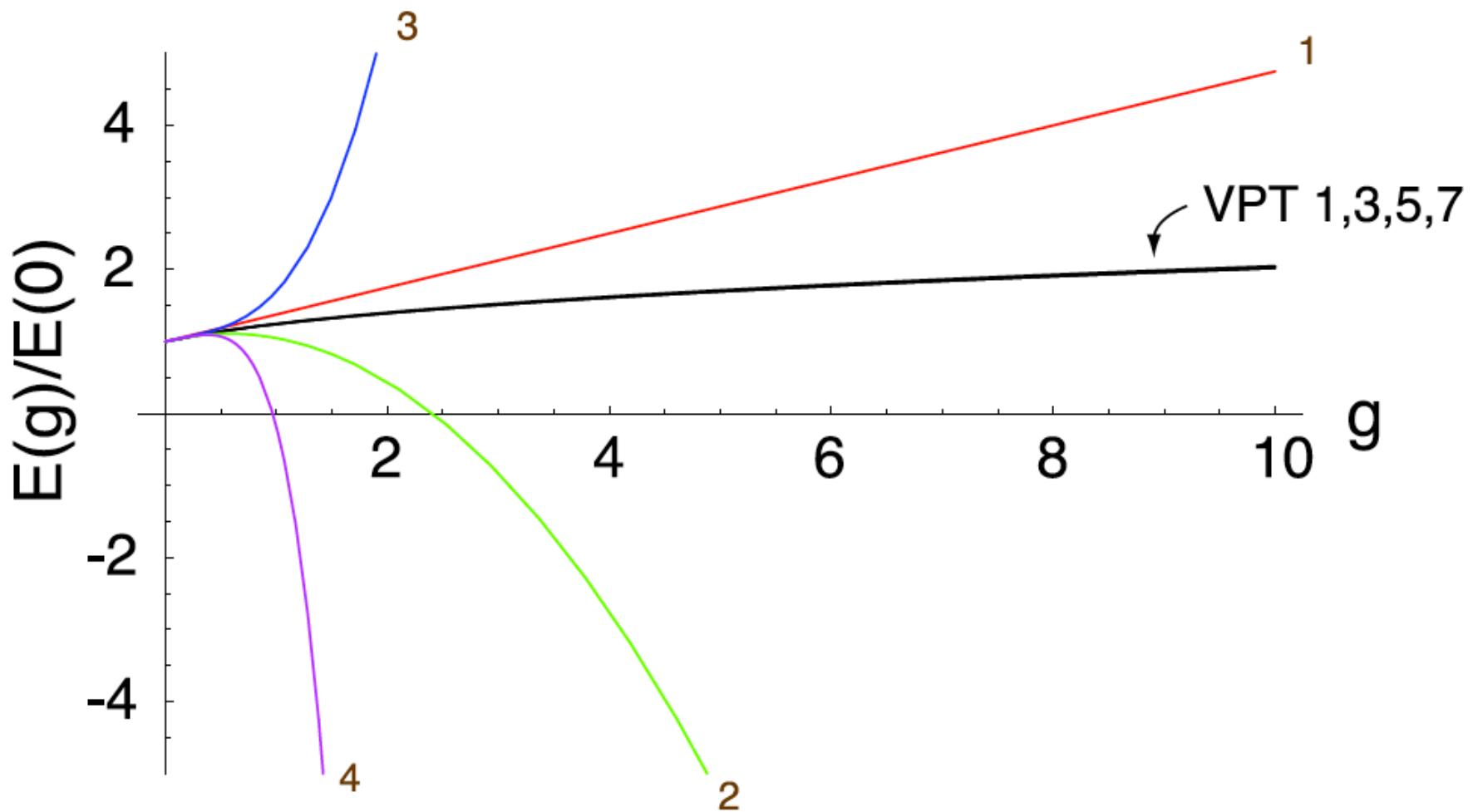
- The coefficients c_n can be computed analytically in this case

$$c_n(r) = \sum_{j=0}^n c_j^{\text{BW}} \binom{(1-3j)/2}{n-j} (2r\Omega)^{n-j}$$

- Fix Ω by imposing variational condition that ground state energy is minimized

$$\frac{\partial E_N}{\partial \Omega} \Big|_{\Omega=\Omega_N} = 0 \quad \rightarrow \text{“Gap equation”}$$

Variational Perturbation Theory



Hard Thermal Loop Perturbation Theory (HTLpt)

High temperature scale separation

At very high temperature there is a hierarchy of length scales in the QGP:

- **Hard Scale, $\lambda \sim 1/T$**
 - $n_b(E) g^2(T) \sim g^2(T)$
 - Wavelength of thermal fluctuations
 - Inverse mass of non-static field modes ($p_0 \neq 0$)
 - Purely perturbative contribution to QCD thermodynamics (g^{2n})
- **Electric Scale, $\lambda \sim 1/gT$**
 - $n_b(E) g^2(T) \sim g(T)$
 - Screening scale for static chromoelectric fluctuations
 - Inverse Debye mass of the A_0
 - Resummation of an infinite subset of diagrams necessary
 - Odd powers of g and logarithms (e.g. $g^3, g^4 \log g$, etc)
- **Magnetic Scale, $\lambda \sim 1/g^2 T$**
 - $n_b(E) g^2(T) \sim g^0(T)$
 - Screening scale for static chromomagnetic fluctuations
 - Inverse “magnetic mass”
 - Generates non-perturbative contribution to pressure starting at 4-loop order (“Linde Problem”)

Hard Thermal Loops

In a high temperature system we must resum a certain class of diagrams which have hard internal (loop) momentum $p_{\text{hard}} \sim T$ and soft external momentum $p_{\text{soft}} \sim gT$

$$\text{---} \circlearrowleft \Pi \circlearrowright \text{---} \cong \left(\text{---} \circlearrowleft \text{---} + \text{---} \circlearrowleft \text{---} + \text{---} \circlearrowleft \text{---} \right) g^2 T^2$$

$$\Pi_T(\omega, p) = \frac{m_D^2}{2} \frac{\omega^2}{p^2} \left[1 + \frac{p^2 - \omega^2}{2\omega p} \log \frac{\omega + p}{\omega - p} \right]$$

$$\Pi_L(\omega, p) = m_D^2 \left[1 - \frac{\omega}{2p} \log \frac{\omega + p}{\omega - p} \right]$$

$$\lim_{\omega \rightarrow 0} \Pi_T(\omega, p) = 0$$

$$\lim_{\omega \rightarrow 0} \Pi_L(\omega, p) = m_D^2$$

At finite temperature there are transverse and longitudinal gluons

$$\Delta_T(p) = \frac{1}{p^2 - \Pi_T(p)}$$

$$\Delta_L(p) = \frac{1}{\mathbf{p}^2 + \Pi_L(p)}$$

Gluons acquire a temperature dependent mass which is proportional to the temperature. At LO one has

$$m_D^2 = \frac{1}{3} \left(N_c + \frac{1}{2} N_f \right) g^2 T^2$$

HTLpt Reorganization

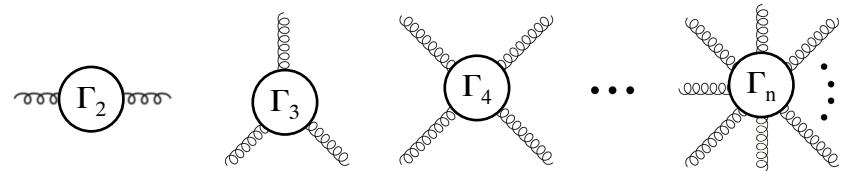
- Reorganize the perturbative calculation by shifting the expansion point for the loop expansion to the high T limit using HTLs
- Expansion parameter δ counts number of dressed loops (+ insertions)
- Reproduces perturbative expansion order-by-order if expanded in a power series in the g
- Resummed Result is all orders in g

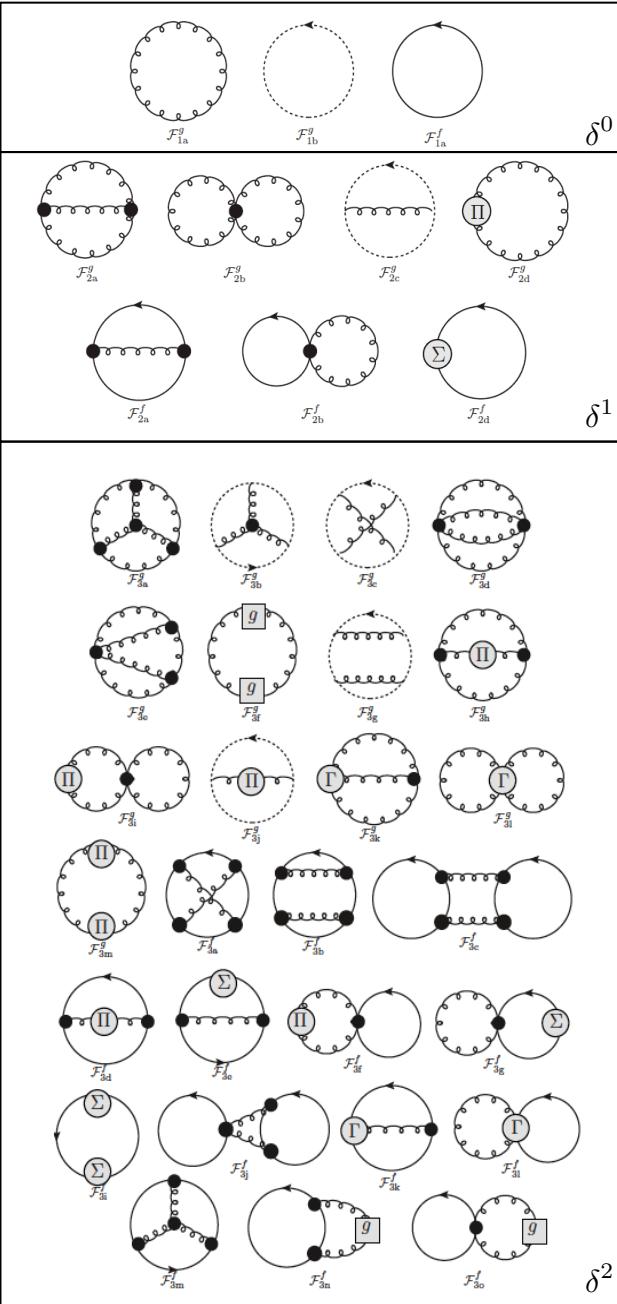
$$\mathcal{L} = (\mathcal{L}_{\text{QCD}} + \mathcal{L}_{\text{HTL}}) \Big|_{g_s \rightarrow \sqrt{\delta} g_s} + \Delta \mathcal{L}_{\text{HTL}}$$

[Andersen, Braaten, and MS, 99 → 02]

$$\begin{aligned} \mathcal{L}_{\text{QCD}} = & -\frac{1}{2} \text{Tr} [G_{\mu\nu} G^{\mu\nu}] + i\bar{\psi}\gamma^\mu D_\mu \psi \\ & + \mathcal{L}_{\text{gf}} + \mathcal{L}_{\text{gh}} + \Delta \mathcal{L}_{\text{QCD}} \end{aligned}$$

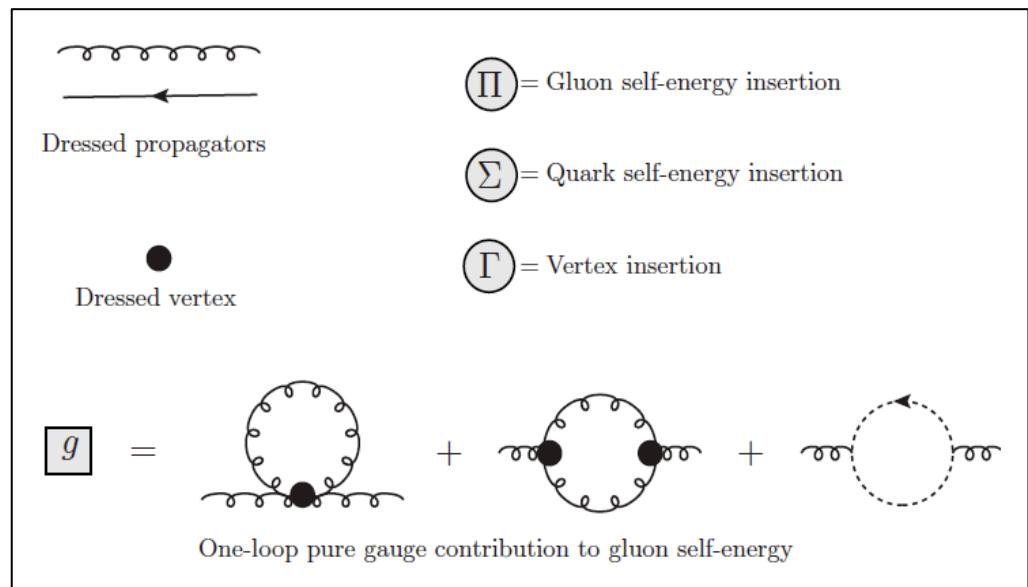
$$\begin{aligned} \mathcal{L}_{\text{HTL}} = & -\frac{1}{2}(1-\delta)m_D^2 \text{Tr} \left(G_{\mu\alpha} \left\langle \frac{y^\alpha y^\beta}{(y \cdot D)^2} \right\rangle_y G^\mu{}_\beta \right) \\ & +(1-\delta)im_q^2 \bar{\psi}\gamma^\mu \left\langle \frac{y_\mu}{y \cdot D} \right\rangle_y \psi, \end{aligned}$$





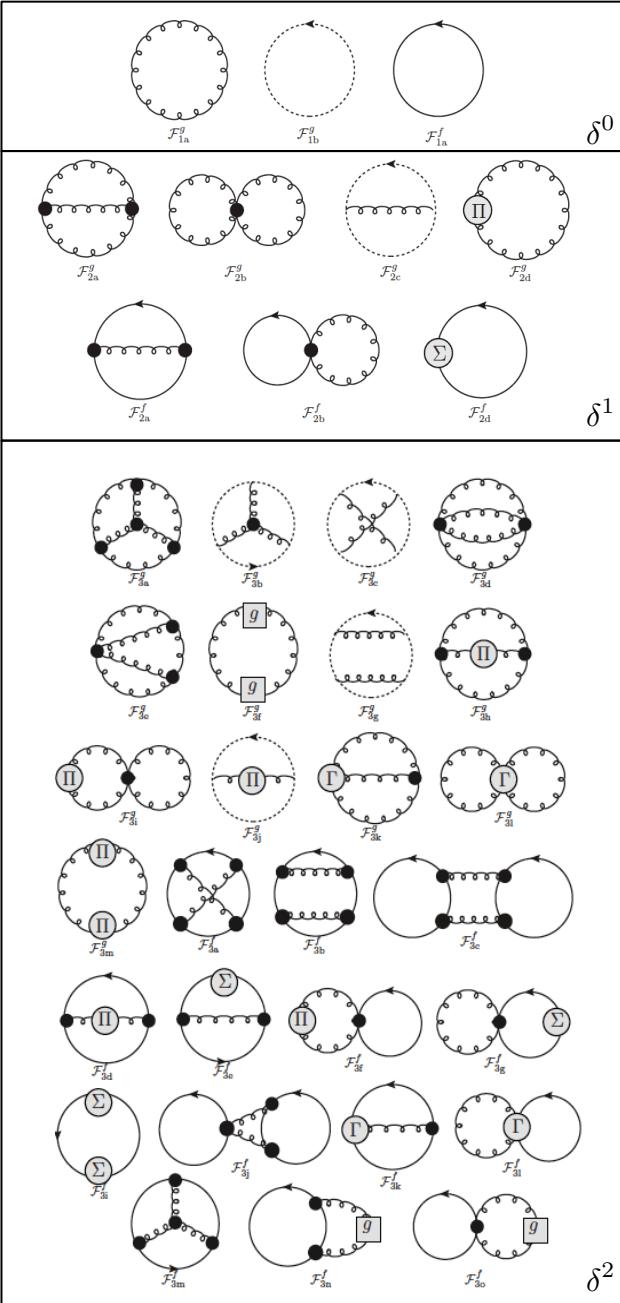
3-loop Calculation

- Now “simply” compute all contributions up to three loops (49 diagrams) using HTL-dressed propagators and vertices.
- Finite T and zero quark chemical potential 3-loop result: Andersen, Leganger, Su, and MS, 1009.4644, 1103.2528



Finite μ and T Result

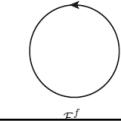
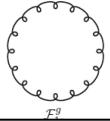
N. Haque, J.O. Andersen, M.G. Mustafa, MS, N. Su, 1309.3968



$$\begin{aligned} \frac{\Omega_{\text{NNLO}}}{\Omega_0} = & \frac{7}{4} \frac{d_F}{d_A} \left(1 + \frac{120}{7} \hat{\mu}^2 + \frac{240}{7} \hat{\mu}^4 \right) + \frac{s_F \alpha_s}{\pi} \left[-\frac{5}{8} (1 + 12\hat{\mu}^2) (5 + 12\hat{\mu}^2) + \frac{15}{2} (1 + 12\hat{\mu}^2) \hat{m}_D \right. \\ & + \frac{15}{2} \left(2 \ln \frac{\hat{\Lambda}_q}{2} - 1 - \aleph(z) \right) \hat{m}_D^3 - 90 \hat{m}_q^2 \hat{m}_D \Big] + s_{2F} \left(\frac{\alpha_s}{\pi} \right)^2 \left[\frac{15}{64} \left\{ 35 - 32 (1 - 12\hat{\mu}^2) \frac{\zeta'(-1)}{\zeta(-1)} + 472 \hat{\mu}^2 \right. \right. \\ & + 1328 \hat{\mu}^4 + 64 \left(-36 i \hat{\mu} \aleph(2, z) + 6(1 + 8\hat{\mu}^2) \aleph(1, z) + 3i \hat{\mu} (1 + 4\hat{\mu}^2) \aleph(0, z) \right) \Big\} - \frac{45}{2} \hat{m}_D (1 + 12\hat{\mu}^2) \Big] \\ & + \left(\frac{s_F \alpha_s}{\pi} \right)^2 \left[\frac{5}{4\hat{m}_D} (1 + 12\hat{\mu}^2)^2 + 30 (1 + 12\hat{\mu}^2) \frac{\hat{m}_q^2}{\hat{m}_D} + \frac{25}{12} \left\{ \left(1 + \frac{72}{5} \hat{\mu}^2 + \frac{144}{5} \hat{\mu}^4 \right) \ln \frac{\hat{\Lambda}_q}{2} \right. \right. \\ & + \frac{1}{20} (1 + 168\hat{\mu}^2 + 2064\hat{\mu}^4) + \frac{3}{5} (1 + 12\hat{\mu}^2)^2 \gamma_E - \frac{8}{5} (1 + 12\hat{\mu}^2) \frac{\zeta'(-1)}{\zeta(-1)} - \frac{34}{25} \frac{\zeta'(-3)}{\zeta(-3)} \\ & - \frac{72}{5} [8\aleph(3, z) + 3\aleph(3, 2z) - 12\hat{\mu}^2 \aleph(1, 2z) + 12i\hat{\mu} (\aleph(2, z) + \aleph(2, 2z)) - i\hat{\mu} (1 + 12\hat{\mu}^2) \aleph(0, z) \\ & - 2(1 + 8\hat{\mu}^2) \aleph(1, z)] \Big\} - \frac{15}{2} (1 + 12\hat{\mu}^2) \left(2 \ln \frac{\hat{\Lambda}_q}{2} - 1 - \aleph(z) \right) \hat{m}_D \Big] \\ & + \left(\frac{c_A \alpha_s}{3\pi} \right) \left(\frac{s_F \alpha_s}{\pi} \right) \left[\frac{15}{2\hat{m}_D} (1 + 12\hat{\mu}^2) - \frac{235}{16} \left\{ \left(1 + \frac{792}{47} \hat{\mu}^2 + \frac{1584}{47} \hat{\mu}^4 \right) \ln \frac{\hat{\Lambda}_q}{2} - \frac{144}{47} (1 + 12\hat{\mu}^2) \ln \hat{m}_D \right. \right. \\ & + \frac{319}{940} \left(1 + \frac{2040}{319} \hat{\mu}^2 + \frac{38640}{319} \hat{\mu}^4 \right) - \frac{24\gamma_E}{47} (1 + 12\hat{\mu}^2) - \frac{44}{47} \left(1 + \frac{156}{11} \hat{\mu}^2 \right) \frac{\zeta'(-1)}{\zeta(-1)} - \frac{268}{235} \frac{\zeta'(-3)}{\zeta(-3)} \\ & - \frac{72}{47} [4i\hat{\mu} \aleph(0, z) + (5 - 92\hat{\mu}^2) \aleph(1, z) + 144i\hat{\mu} \aleph(2, z) + 52\aleph(3, z)] \Big\} + 90 \frac{\hat{m}_q^2}{\hat{m}_D} + \frac{315}{4} \left\{ \left(1 + \frac{132}{7} \hat{\mu}^2 \right) \ln \frac{\hat{\Lambda}_q}{2} \right. \\ & \left. \left. + \frac{11}{7} (1 + 12\hat{\mu}^2) \gamma_E + \frac{9}{14} \left(1 + \frac{132}{9} \hat{\mu}^2 \right) + \frac{2}{7} \aleph(z) \right\} \hat{m}_D \right] + \frac{\Omega_{\text{NNLO}}^{\text{YM}}(\Lambda_g)}{\Omega_0}. \end{aligned}$$

$$\begin{aligned} \hat{m}_D^2 = & \frac{\alpha_s}{3\pi} \left\{ c_A + \frac{c_A^2 \alpha_s}{12\pi} \left(5 + 22\gamma_E + 22 \ln \frac{\hat{\Lambda}_g}{2} \right) + s_F (1 + 12\hat{\mu}^2) + \frac{c_A s_F \alpha_s}{12\pi} ((9 + 132\hat{\mu}^2) + 22 (1 + 12\hat{\mu}^2) \gamma_E \right. \\ & \left. + 2 (7 + 132\hat{\mu}^2) \ln \frac{\hat{\Lambda}_q}{2} + 4\aleph(z)) + \frac{s_F^2 \alpha_s}{3\pi} (1 + 12\hat{\mu}^2) \left(1 - 2 \ln \frac{\hat{\Lambda}_q}{2} + \aleph(z) \right) - \frac{3}{2} \frac{s_F \alpha_s}{\pi} (1 + 12\hat{\mu}^2) \right\} \end{aligned}$$

Use one-loop running with $\alpha_s(1.5 \text{ GeV}) = 0.326$ taken from state-of-the-art lattice measurement (Bazakov et al 12)


 δ^0

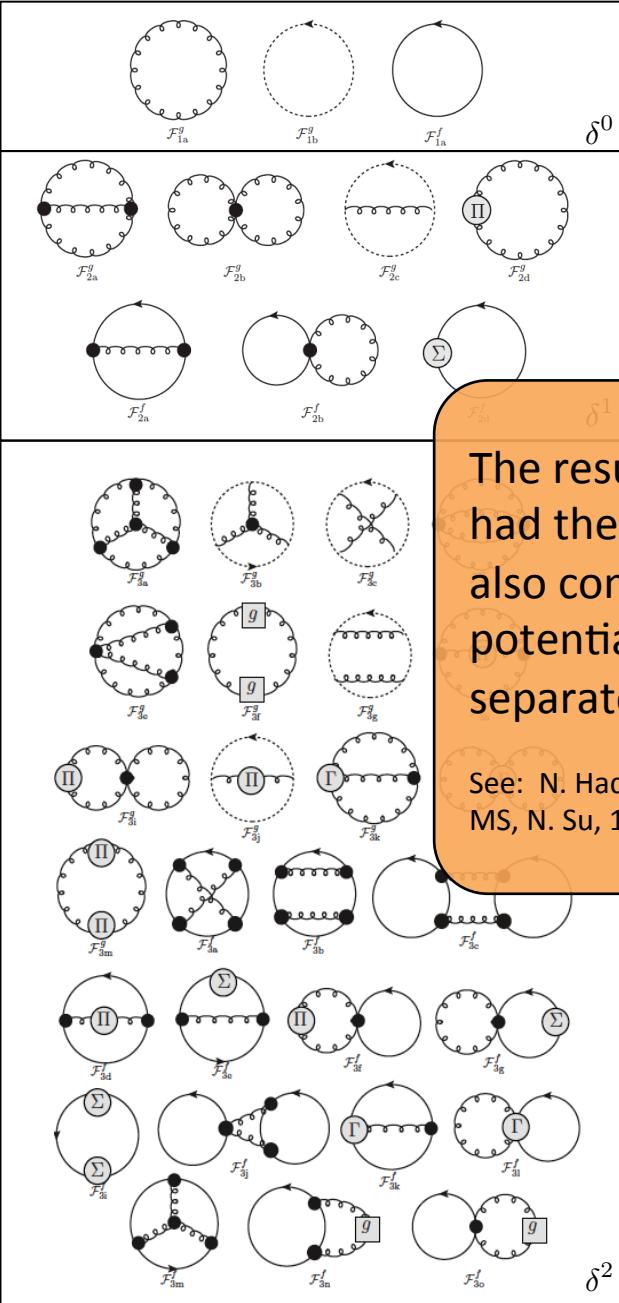
Finite μ and T Result

N. Haque, J.O. Andersen, M.G. Mustafa, MS, N. Su, 1309.3968

$$\begin{aligned}
\frac{\Omega_{\text{NNLO}}}{\Omega_0} = & \frac{7}{4} \frac{d_F}{d_A} \left(1 + \frac{120}{7} \hat{\mu}^2 + \frac{240}{7} \hat{\mu}^4 \right) + \frac{s_F \alpha_s}{\pi} \left[-\frac{5}{8} (1 + 12\hat{\mu}^2) (5 + 12\hat{\mu}^2) + \frac{15}{2} (1 + 12\hat{\mu}^2) \hat{m}_D \right. \\
& + \frac{15}{2} \left(2 \ln \frac{\hat{\Lambda}_q}{2} - 1 - \aleph(z) \right) \hat{m}_D^3 - 90 \hat{m}_q^2 \hat{m}_D \Big] + s_{2F} \left(\frac{\alpha_s}{\pi} \right)^2 \left[\frac{15}{64} \left\{ 35 - 32 (1 - 12\hat{\mu}^2) \frac{\zeta'(-1)}{\zeta(-1)} + 472 \hat{\mu}^2 \right. \right. \\
& + 1328 \hat{\mu}^4 + 64 \left(-36 i \hat{\mu} \aleph(2, z) + 6(1 + 8\hat{\mu}^2) \aleph(1, z) + 3i \hat{\mu} (1 + 4\hat{\mu}^2) \aleph(0, z) \right) \Big\} - \frac{45}{2} \hat{m}_D (1 + 12\hat{\mu}^2) \Big] \\
& + \left(\frac{s_F \alpha_s}{\pi} \right)^2 \left[\frac{5}{4 \hat{m}_D} (1 + 12\hat{\mu}^2)^2 + 30 (1 + 12\hat{\mu}^2) \frac{\hat{m}_q^2}{\hat{m}_D} + \frac{25}{12} \left\{ \left(1 + \frac{72}{5} \hat{\mu}^2 + \frac{144}{5} \hat{\mu}^4 \right) \ln \frac{\hat{\Lambda}_q}{2} \right. \right. \\
& + \frac{1}{20} (1 + 168\hat{\mu}^2 + 2064\hat{\mu}^4) + \frac{3}{5} (1 + 12\hat{\mu}^2)^2 \gamma_E - \frac{8}{5} (1 + 12\hat{\mu}^2) \frac{\zeta'(-1)}{\zeta(-1)} - \frac{34}{25} \frac{\zeta'(-3)}{\zeta(-3)} \\
& - \frac{72}{5} \left[8\aleph(3, z) + 3\aleph(3, 2z) - 12\hat{\mu}^2 \aleph(1, 2z) + 12i \hat{\mu} (\aleph(2, z) + \aleph(2, 2z)) - i \hat{\mu} (1 + 12\hat{\mu}^2) \aleph(0, z) \right. \\
& \left. \left. - 2(1 + 8\hat{\mu}^2) \aleph(1, z) \right] \right\} - \frac{15}{2} (1 + 12\hat{\mu}^2) \left(2 \ln \frac{\hat{\Lambda}_q}{2} - 1 - \aleph(z) \right) \hat{m}_D \Big] \\
& + \left(\frac{c_A \alpha_s}{3\pi} \right) \left(\frac{s_F \alpha_s}{\pi} \right) \left[\frac{15}{2 \hat{m}_D} (1 + 12\hat{\mu}^2) - \frac{235}{16} \left\{ \left(1 + \frac{792}{47} \hat{\mu}^2 + \frac{1584}{47} \hat{\mu}^4 \right) \ln \frac{\hat{\Lambda}_q}{2} - \frac{144}{47} (1 + 12\hat{\mu}^2) \ln \hat{m}_D \right. \right. \\
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& - \frac{72}{47} \left[4i \hat{\mu} \aleph(0, z) + (5 - 92\hat{\mu}^2) \aleph(1, z) + 144i \hat{\mu} \aleph(2, z) + 52\aleph(3, z) \right] \Big\} + 90 \frac{\hat{m}_q^2}{\hat{m}_D} + \frac{315}{4} \left\{ \left(1 + \frac{132}{7} \hat{\mu}^2 \right) \ln \frac{\hat{\Lambda}_q}{2} \right. \\
& \left. \left. + \frac{11}{7} (1 + 12\hat{\mu}^2) \gamma_E + \frac{9}{14} \left(1 + \frac{132}{9} \hat{\mu}^2 \right) + \frac{2}{7} \aleph(z) \right\} \hat{m}_D \Big] + \frac{\Omega_{\text{NNLO}}^{\text{YM}}(\Lambda_g)}{\Omega_0}. \quad (5)
\end{aligned}$$

Finite μ and T Result

N. Haque, J.O. Andersen, M.G. Mustafa, MS, N. Su, 1309.3968



The result below assumed that all quarks had the same chemical potential. We have also computed the thermodynamic potential allowing each quark to have a separate chemical potential.

See: N. Haque, A. Bandyopadhyay, J.O. Andersen, M.G. Mustafa, MS, N. Su, 1402.6907

$$\begin{aligned}
 \frac{\Omega_{\text{NNLO}}}{\Omega_0} = & \frac{7}{4} \frac{d_F}{d_A} \left(1 + \frac{120}{7} \hat{\mu}^2 + \frac{240}{7} \hat{\mu}^4 \right) + \frac{s_F \alpha_s}{\pi} \left[-\frac{5}{8} (1+12\hat{\mu}^2) (5+12\hat{\mu}^2) + \frac{15}{2} (1+12\hat{\mu}^2) \hat{m}_D \right. \\
 & + \frac{15}{2} \left(2 \ln \frac{\hat{\Lambda}_q}{2} - 1 - \aleph(z) \right) \hat{m}_D^3 - 90 \hat{m}_q^2 \hat{m}_D \Big] + s_{2F} \left(\frac{\alpha_s}{\pi} \right)^2 \left[\frac{15}{64} \left\{ 35 - 32 (1-12\hat{\mu}^2) \frac{\zeta'(-1)}{\zeta(-1)} + 472 \hat{\mu}^2 \right. \right. \\
 & \left. \left. + 1328 \hat{\mu}^4 + 64 \left(-36 i \hat{\mu} \aleph(2, z) + 6(1+8\hat{\mu}^2) \aleph(1, z) + 3i \hat{\mu} (1+4\hat{\mu}^2) \aleph(0, z) \right) \right\} - \frac{45}{2} \hat{m}_D (1+12\hat{\mu}^2) \right] \\
 & + \left(\frac{s_F \alpha_s}{\pi} \right)^2 \left[\frac{5}{4 \hat{m}_D} (1+12\hat{\mu}^2)^2 + 30 (1+12\hat{\mu}^2) \frac{\hat{m}_q^2}{\hat{m}_D} + \frac{25}{12} \right\{ \left(1 + \frac{72}{5} \hat{\mu}^2 + \frac{144}{5} \hat{\mu}^4 \right) \ln \frac{\hat{\Lambda}_q}{2} \right. \\
 & + \frac{1}{20} (1+168\hat{\mu}^2 + 2064\hat{\mu}^4) + \frac{1}{2} (1+12\hat{\mu}^2) \gamma_E - \frac{1}{2} (1+12\hat{\mu}^2) \frac{\zeta'(-1)}{\zeta(-1)} - \frac{34}{25} \frac{\zeta'(-3)}{\zeta(-3)} \\
 & \left. \left. + \frac{1}{2} [8\aleph(3, z) + 3\aleph(3, 2z) - 12\hat{\mu}^2 \aleph(1, 2z) + 12i\hat{\mu} (\aleph(2, z) + \aleph(2, 2z)) - i\hat{\mu} (1+12\hat{\mu}^2) \aleph(0, z) \right. \right. \\
 & \left. \left. - 2(1+8\hat{\mu}^2) \aleph(1, z) \right] \left(\frac{15}{2} (1+12\hat{\mu}^2) \left(2 \ln \frac{\hat{\Lambda}_q}{2} - 1 - \aleph(z) \right) \hat{m}_D \right) \right] \\
 & \left(\frac{10}{3} \right) \left(\frac{80}{3} \right) \left[\frac{5}{2 \hat{m}_D} (1+12\hat{\mu}^2) - \frac{235}{16} \left\{ \left(1 + \frac{792}{47} \hat{\mu}^2 + \frac{1584}{47} \hat{\mu}^4 \right) \ln \frac{\hat{\Lambda}_q}{2} - \frac{144}{47} (1+12\hat{\mu}^2) \ln \hat{m}_D \right. \right. \\
 & + \frac{319}{47} \left(\frac{1+2040}{\hat{\mu}^2} \frac{38640}{\hat{\mu}^4} \right) \frac{24 \gamma_E}{(1+12\hat{\mu}^2)} - \frac{44}{47} \left(1 + \frac{156}{11} \hat{\mu}^2 \right) \frac{\zeta'(-1)}{\zeta(-1)} - \frac{268}{235} \frac{\zeta'(-3)}{\zeta(-3)} \\
 & \left. \left. - \frac{72}{47} [4i\hat{\mu} \aleph(0, z) + (5-92\hat{\mu}^2) \aleph(1, z) + 144i\hat{\mu} \aleph(2, z) + 52\aleph(3, z)] \right\} + 90 \frac{\hat{m}_q^2}{\hat{m}_D} + \frac{315}{4} \left\{ \left(1 + \frac{132}{7} \hat{\mu}^2 \right) \ln \frac{\hat{\Lambda}_q}{2} \right. \right. \\
 & \left. \left. + \frac{11}{7} (1+12\hat{\mu}^2) \gamma_E + \frac{9}{14} \left(1 + \frac{132}{9} \hat{\mu}^2 \right) + \frac{2}{7} \aleph(z) \right\} \hat{m}_D \right] + \frac{\Omega_{\text{NNLO}}^{\text{YM}}(\Lambda_g)}{\Omega_0}.
 \end{aligned}$$

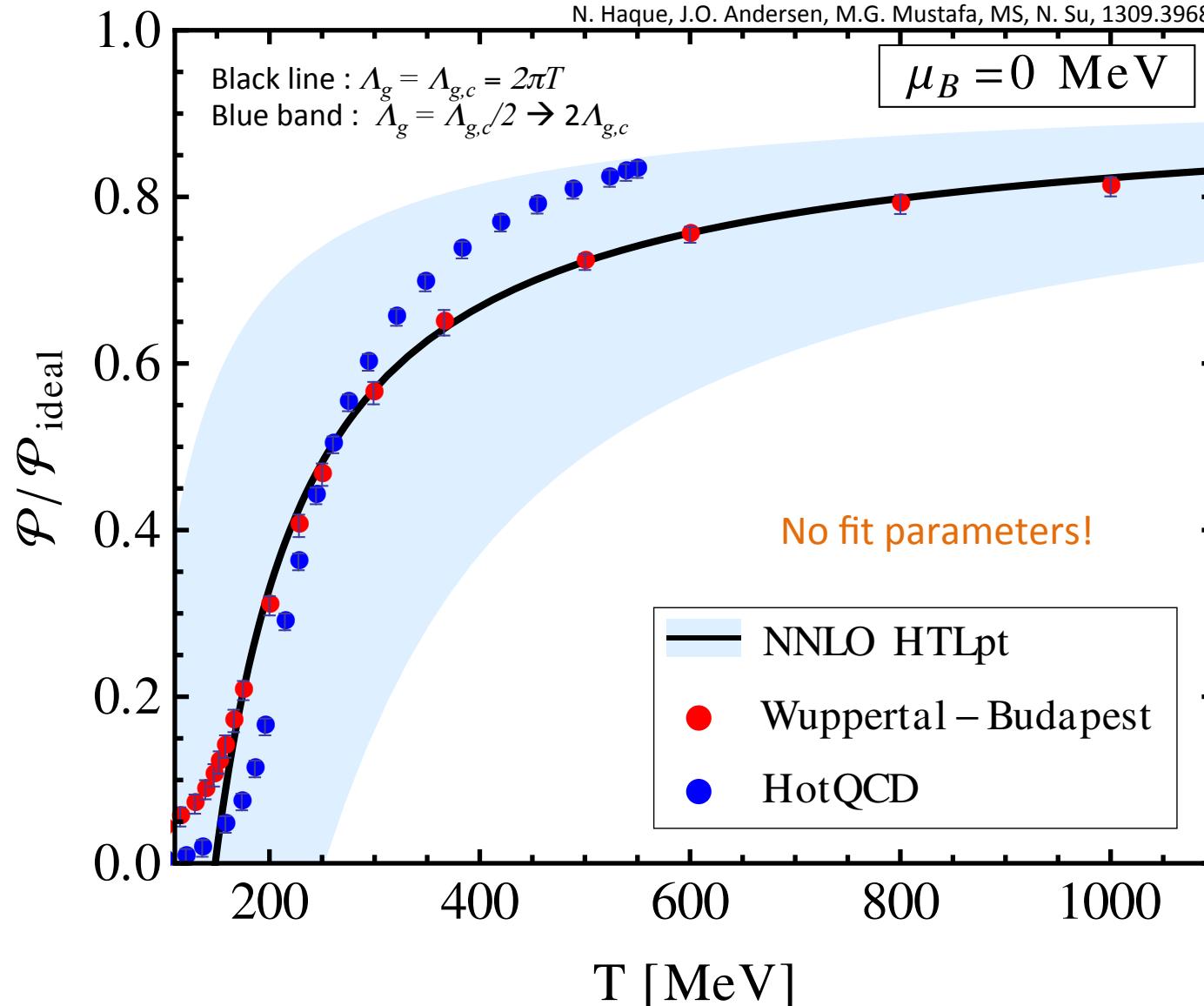
$$\begin{aligned}
 \hat{m}_D^2 = & \frac{\alpha_s}{3\pi} \left\{ c_A + \frac{c_A^2 \alpha_s}{12\pi} \left(5 + 22\gamma_E + 22 \ln \frac{\hat{\Lambda}_q}{2} \right) + s_F (1+12\hat{\mu}^2) + \frac{c_A s_F \alpha_s}{12\pi} ((9+132\hat{\mu}^2) + 22(1+12\hat{\mu}^2) \gamma_E \right. \\
 & \left. + 2(7+132\hat{\mu}^2) \ln \frac{\hat{\Lambda}_q}{2} + 4\aleph(z) \right) + \frac{s_F^2 \alpha_s}{3\pi} (1+12\hat{\mu}^2) \left(1 - 2 \ln \frac{\hat{\Lambda}_q}{2} + \aleph(z) \right) - \frac{3}{2} \frac{s_F \alpha_s}{\pi} (1+12\hat{\mu}^2) \right\}
 \end{aligned}$$

Use one-loop running with $\alpha_s(1.5 \text{ GeV}) = 0.326$ taken from state-of-the-art lattice measurement (Bazakov et al 12)

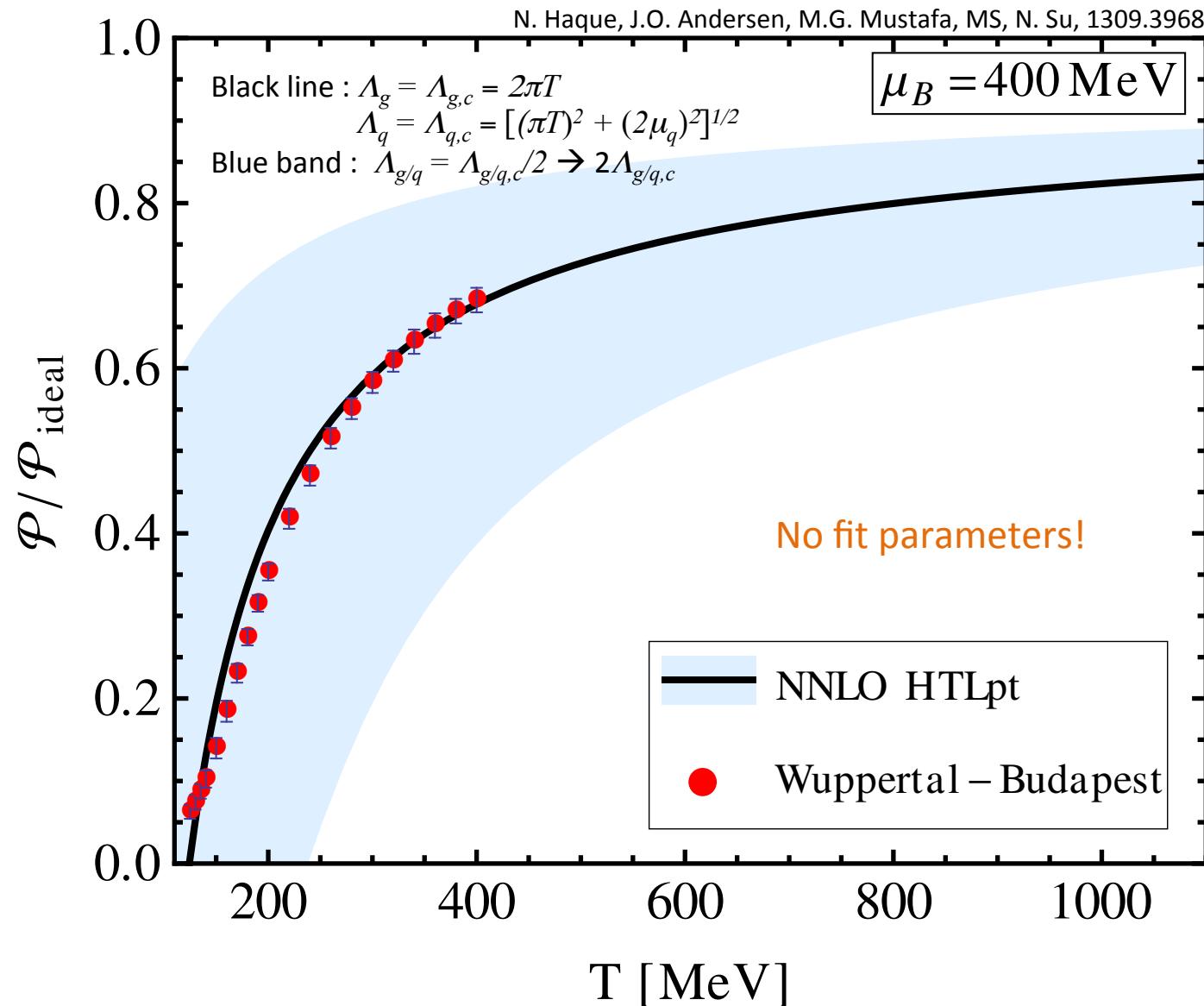
Pressure vs Temperature – $\mu_B = 0$ MeV

Andersen, Leganger, Su, and MS 1009.4644, 1103.2528

N. Haque, J.O. Andersen, M.G. Mustafa, MS, N. Su, 1309.3968

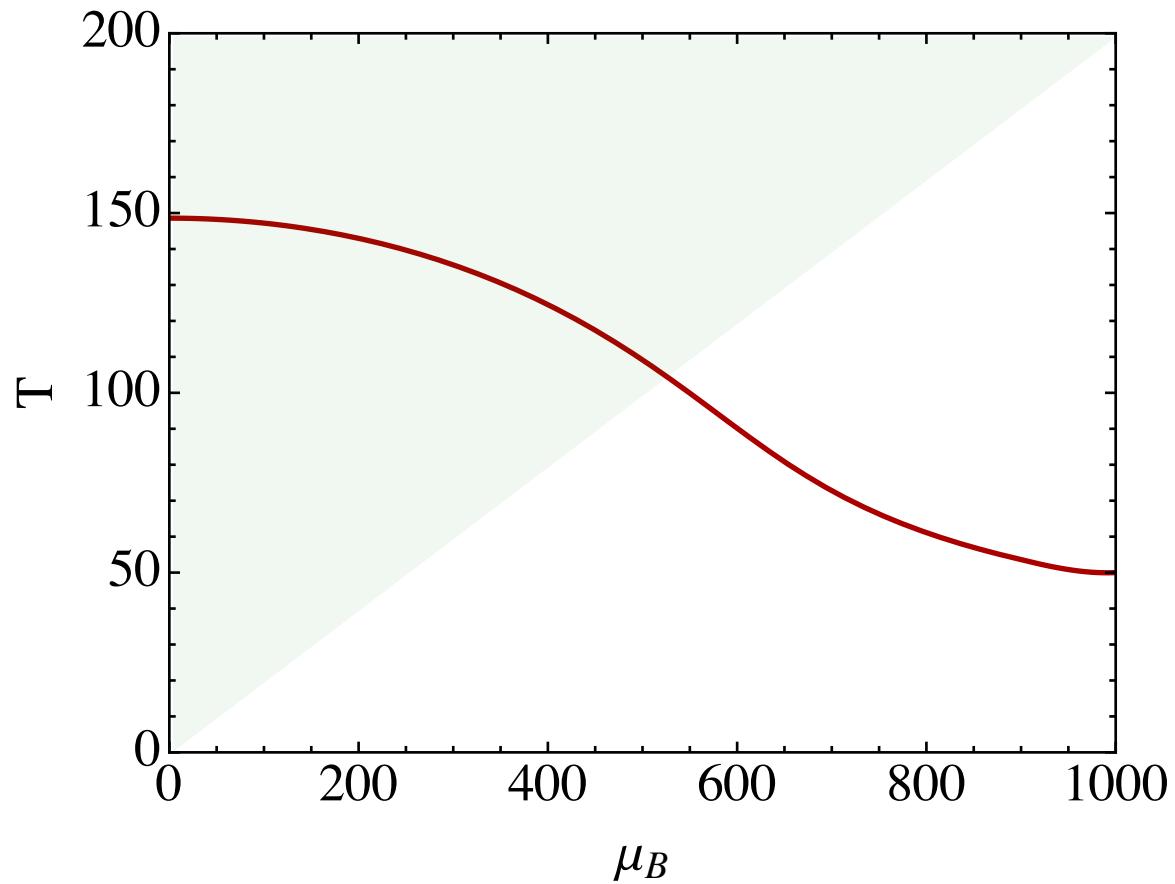
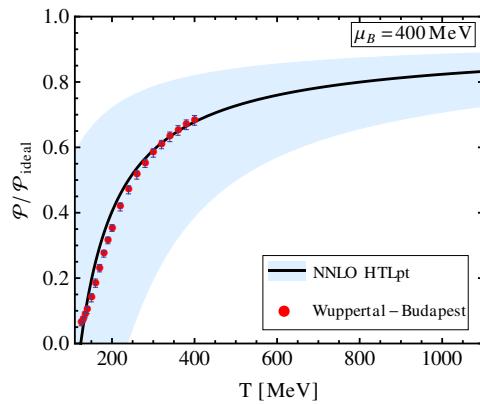
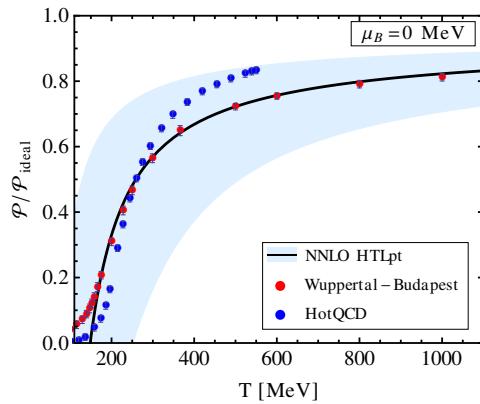


Pressure vs Temperature – $\mu_B = 400$ MeV



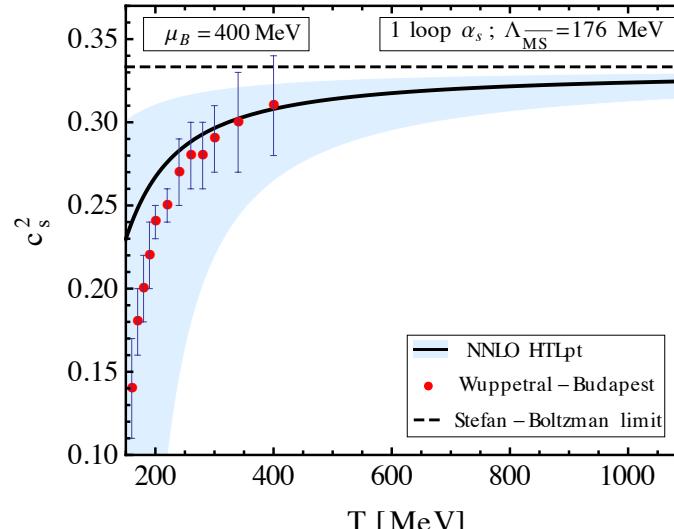
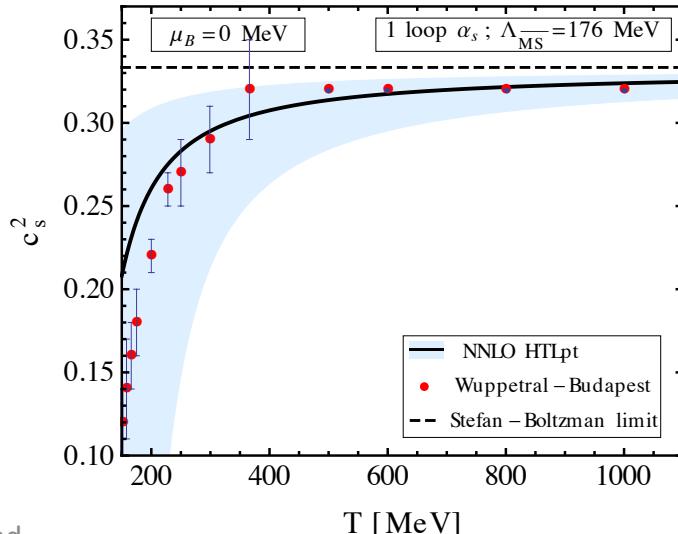
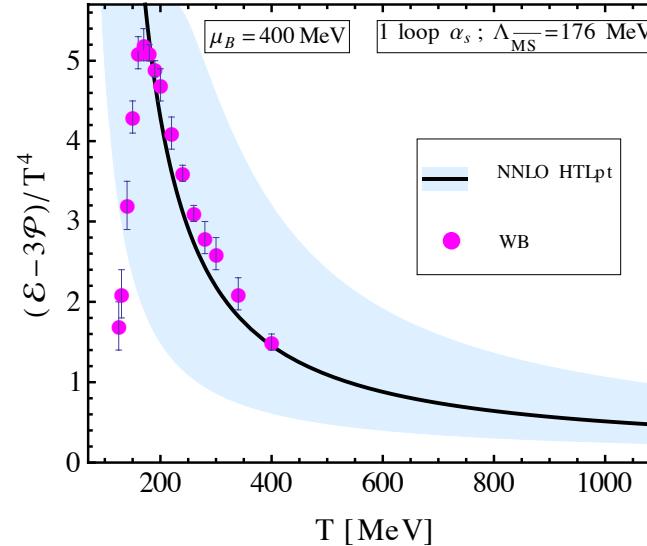
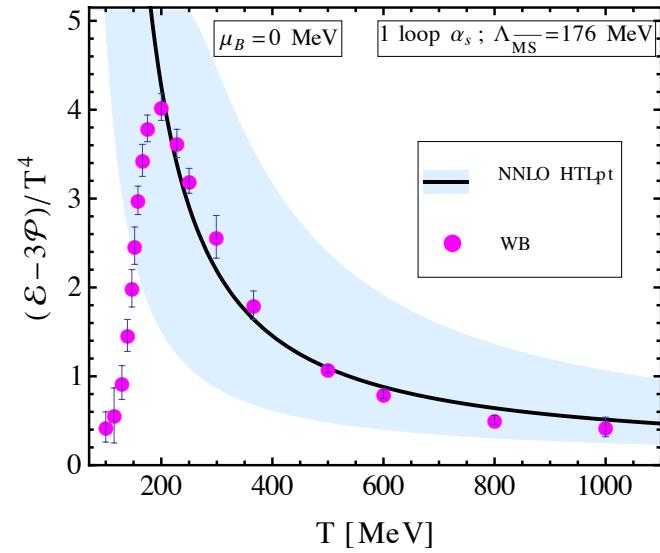
HTLpt “Phase Transition Line”

Although we probably shouldn't do it since we are extrapolating out of our “region of trust,” we can solve for the point where the HTLpt pressure goes to zero in order to extract a kind of phase transition line.



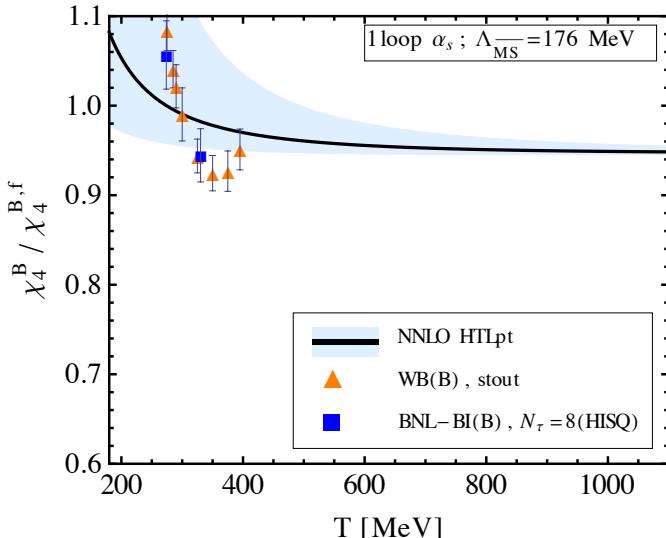
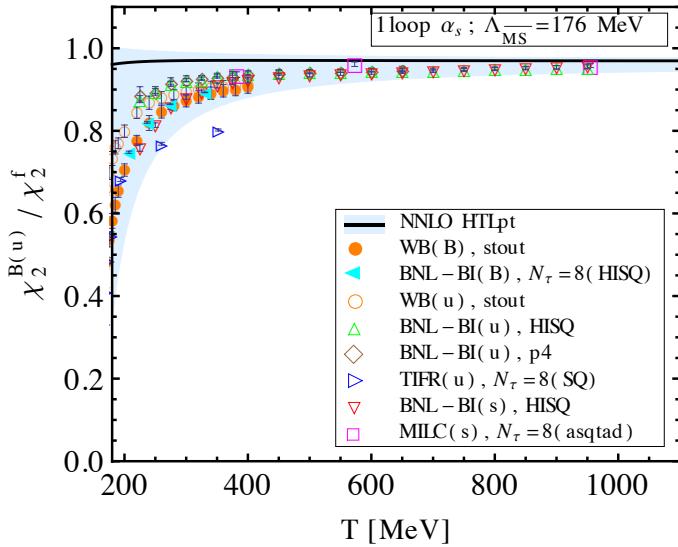
Trace Anomaly and Speed of Sound

N. Haque, A. Bandyopadhyay, J.O. Andersen, M.G. Mustafa, MS, N. Su, 1402.6907



Baryon Number Susceptibilities

N. Haque, A. Bandyopadhyay, J.O. Andersen, M.G. Mustafa, MS, N. Su, 1402.6907



General Quark Susceptibility

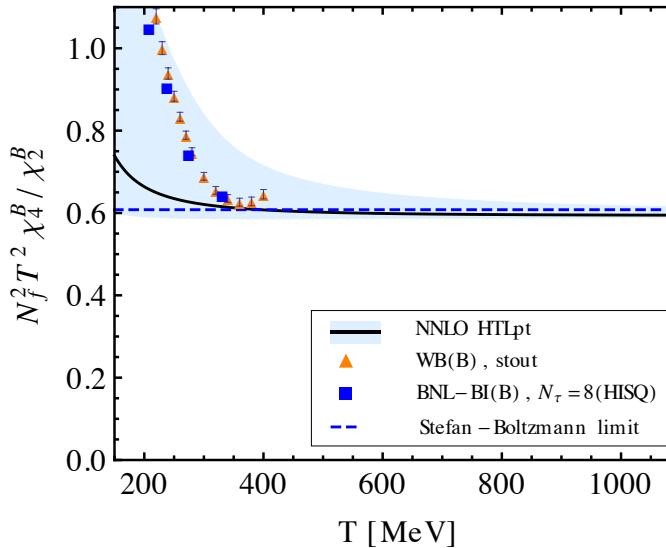
$$\chi_{ijk\dots}(T) \equiv \left. \frac{\partial^{i+j+k+\dots} \mathcal{P}(T, \mu)}{\partial \mu_u^i \partial \mu_d^j \partial \mu_s^k \dots} \right|_{\mu=0}$$

Baryon Number Susceptibility

$$\chi_B^n(T) \equiv \left. \frac{\partial^n \mathcal{P}}{\partial \mu_B^n} \right|_{\mu_B=0}$$

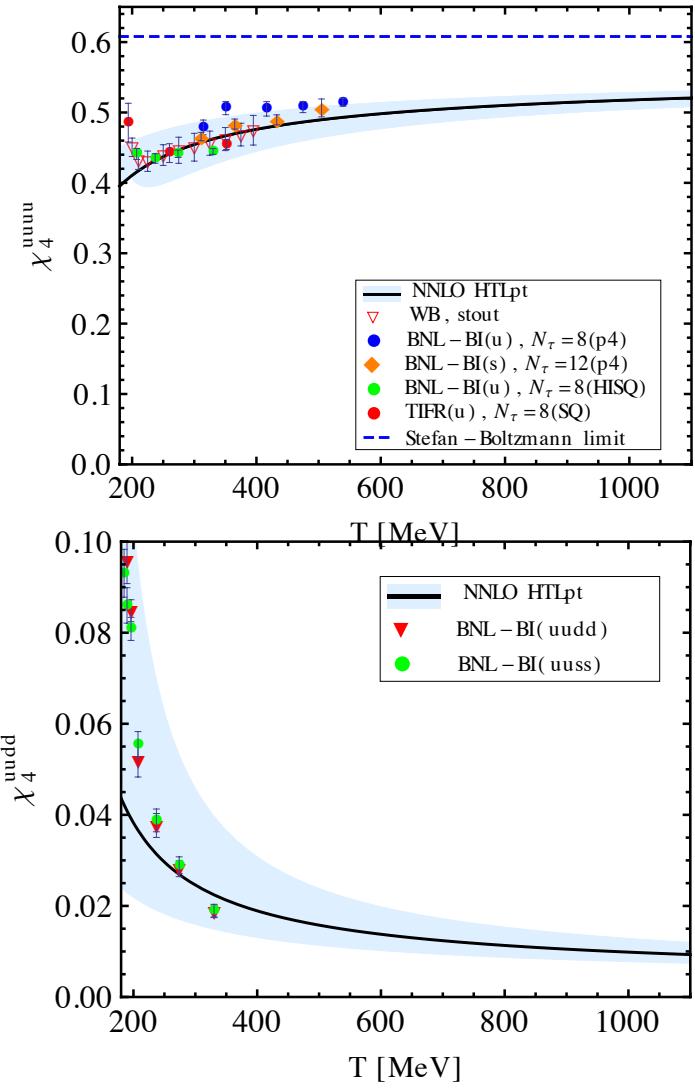
For example

$$\chi_2^B = \frac{1}{9} \left[\chi_2^{uu} + \chi_2^{dd} + \chi_2^{ss} + 2\chi_2^{ud} + 2\chi_2^{ds} + 2\chi_2^{us} \right]$$

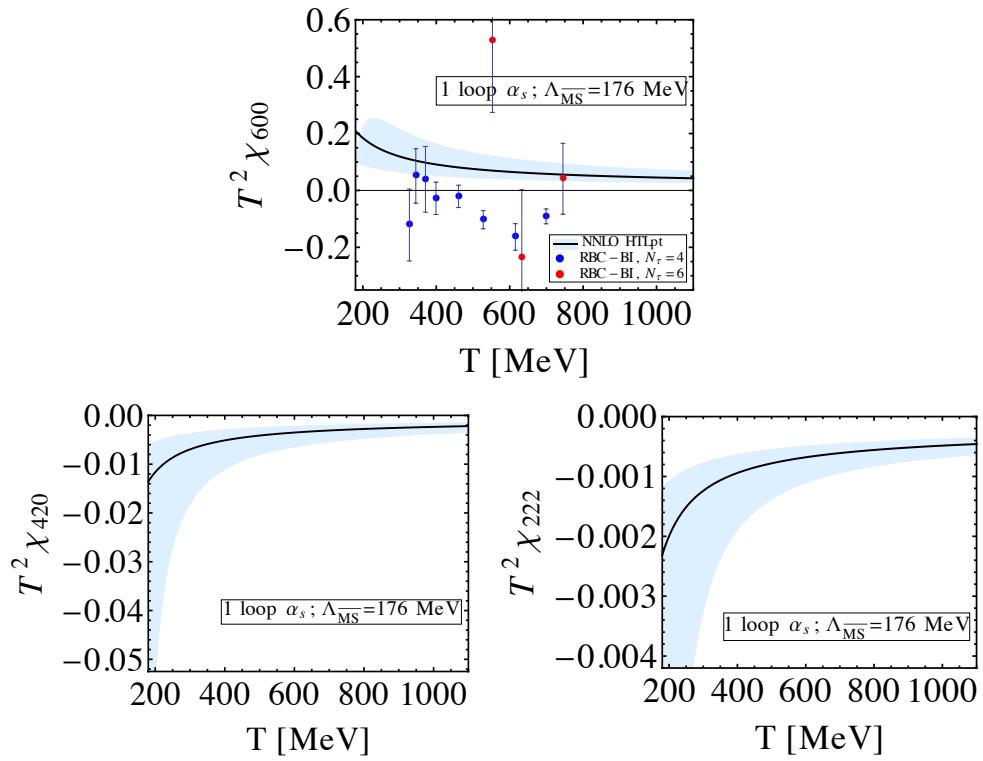


Quark Number Susceptibilities

N. Haque, A. Bandyopadhyay, J.O. Andersen, M.G. Mustafa, MS, N. Su, 1402.6907



We also computed the fourth and sixth-order diagonal and off-diagonal light quark number susceptibilities

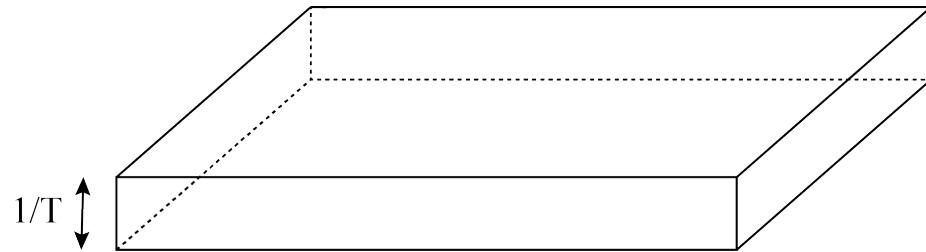


I am not impressed with
your resummed 3-loop
result at all. There is
probably a large
correction at 4 loops and
you will never be able to
calculate it.



Dimensional Reduction

- Scale hierarchy → Integrate out massive (non-static) modes
(Ginsparg 80; Gross, Pisarski, and Yaffe 81; Appelquist and Pisarski, 81)
- Results in effective description for scales $\Delta x \gtrsim 1/gT$



- In the high temperature limit one obtains a 3d effective theory for static electric modes (Braaten and Nieto 1995, Kajantie et al 2003)

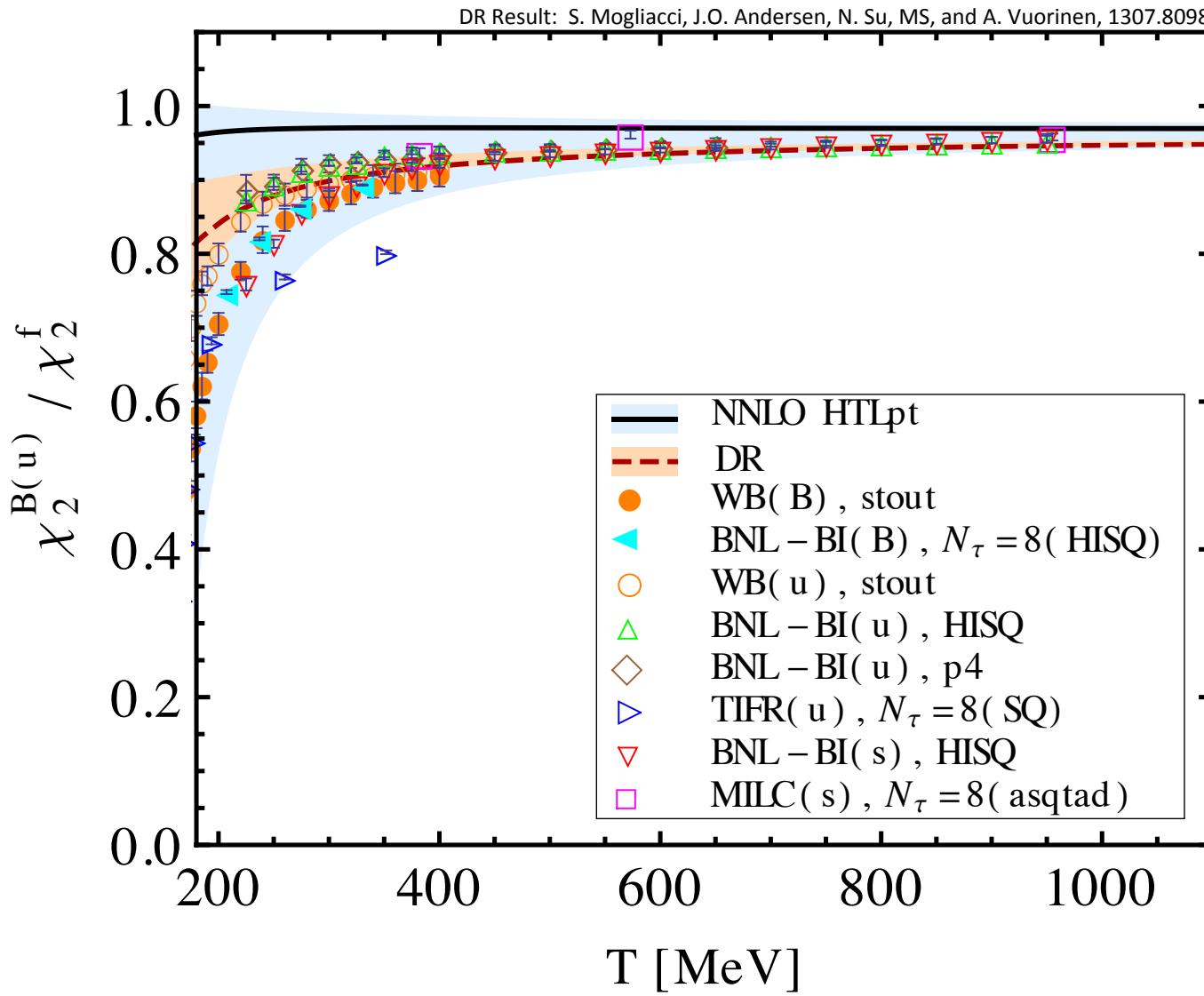
$$\mathcal{L}_{\text{EQCD}} = \frac{1}{g_E^2} \left(\frac{1}{2} \text{Tr}[F_{ij}^2] + \text{Tr}[(D_i A_0)^2] + m_E^2 \text{Tr}[A_0^2] + \lambda_E \text{Tr}[A_0^4] \right) + \delta \mathcal{L}_E$$

$$g_E \equiv \sqrt{T}g, \quad m_E \sim gT, \quad \lambda_E \sim g^2$$

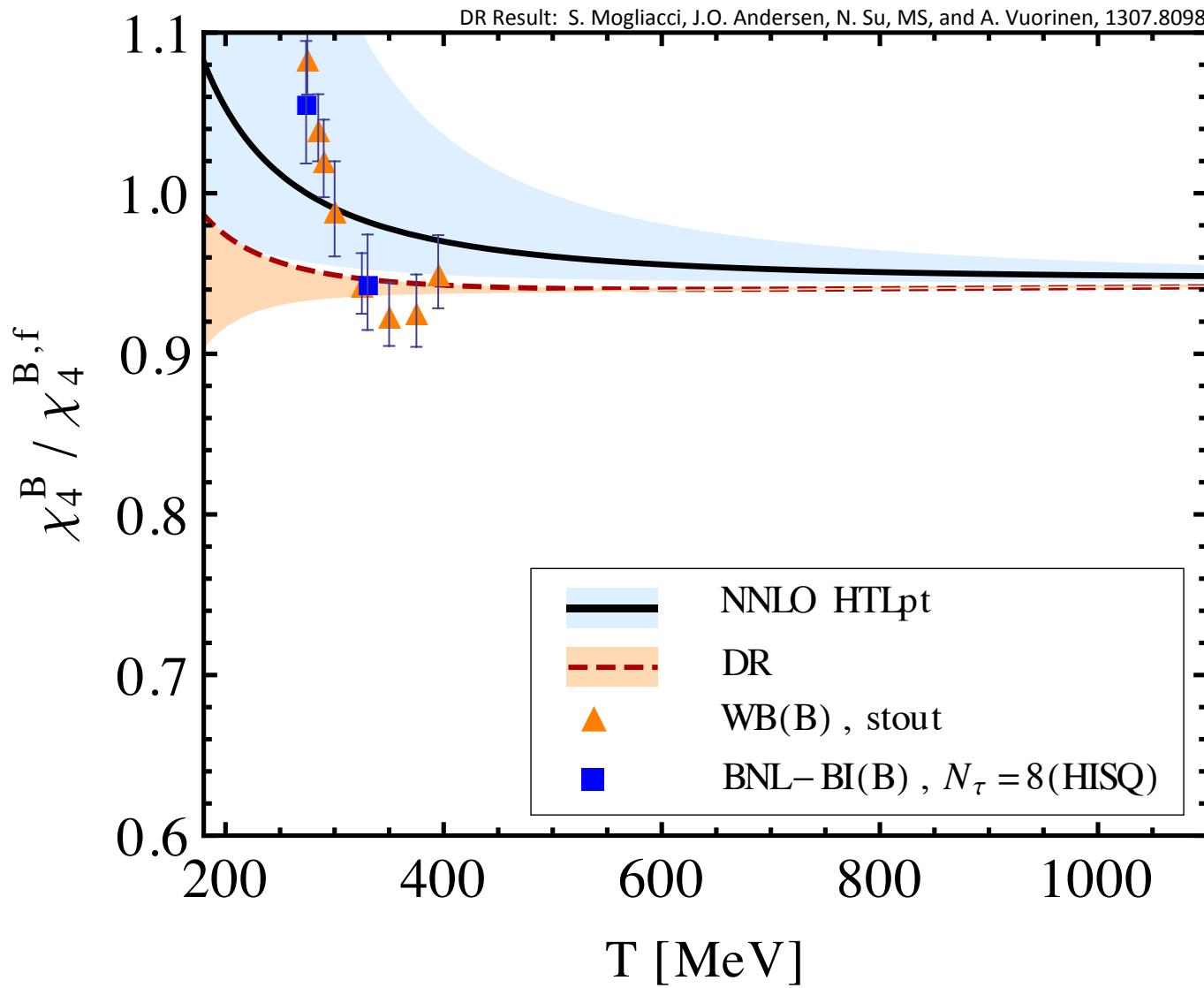
Dim Reduction → Resummed EQCD

- For the pressure, a completely non-perturbative magnetic (MQCD) contribution enters at 4-loop order (only 1 number)
- However, for susceptibilities this number is not needed!
- Hard scale contributions are strictly perturbative
- Soft scale contributions involve inverse powers of m_D
- In order to resum the soft sector contributions, one should not Taylor expand the Debye mass contributions in g , but instead keep the full g -dependence
- Similar to HTLpt, this results in an expression which contains terms of all orders in the strong coupling constant → Resummed EQCD

4-loop Resummed DR Result – $\chi_2^{B,u}$

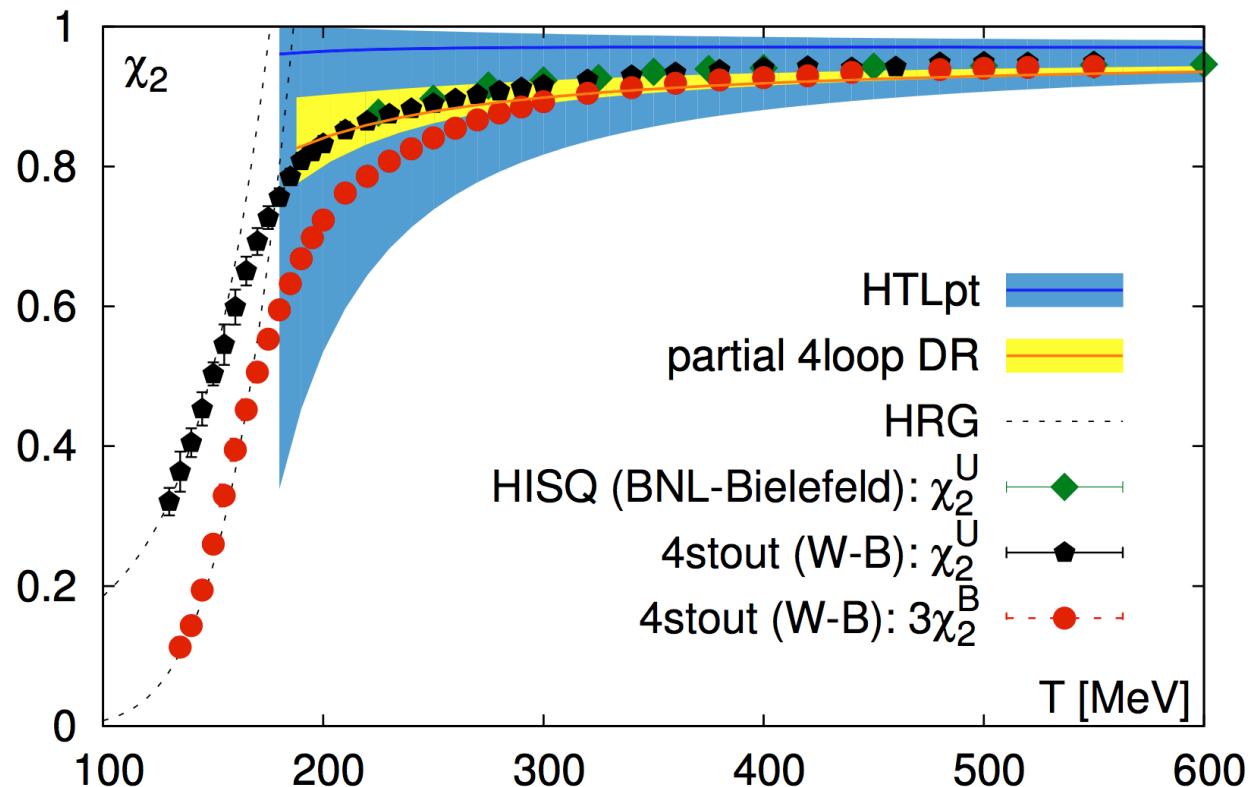


4-loop Resummed DR Result – χ_{4B}



New lattice results

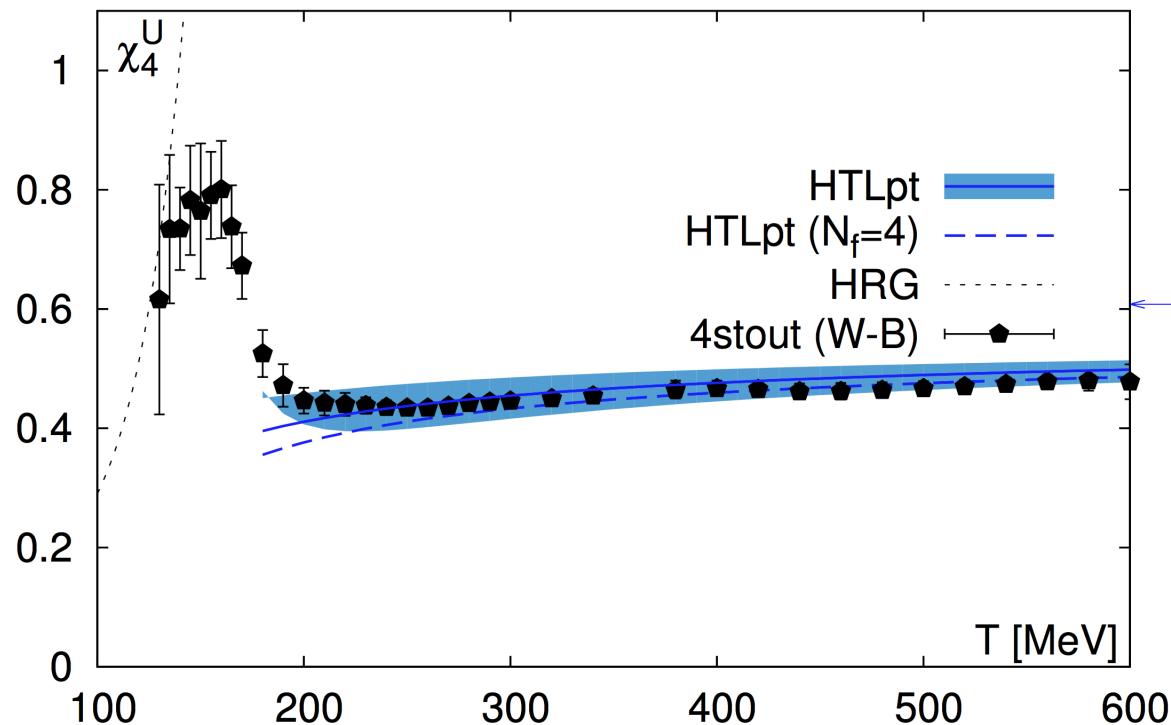
R. Bellwied, S. Borsanyi, Z. Fodor, S. D. Katz, A. Pasztor, C. Ratti,
 K. K. Szabo, 1507.04627



DR = dimensional reduction : S. Mogliacci, J.O. Andersen, MS, N. Su, and A. Vuorinen, 1307.8098

HTLpt = NNLO order : N. Haque, A. Bandyopadhyay, J.O. Andersen, M.G. Mustafa, MS, and N. Su, 1402.6907

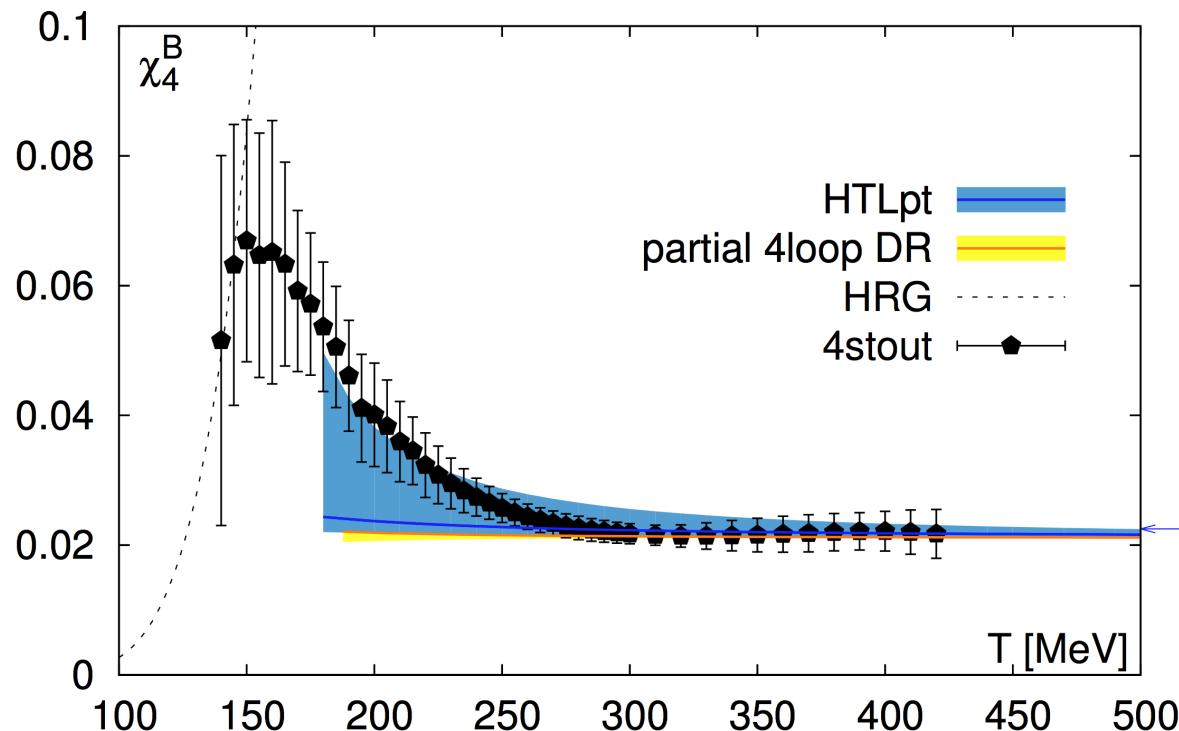
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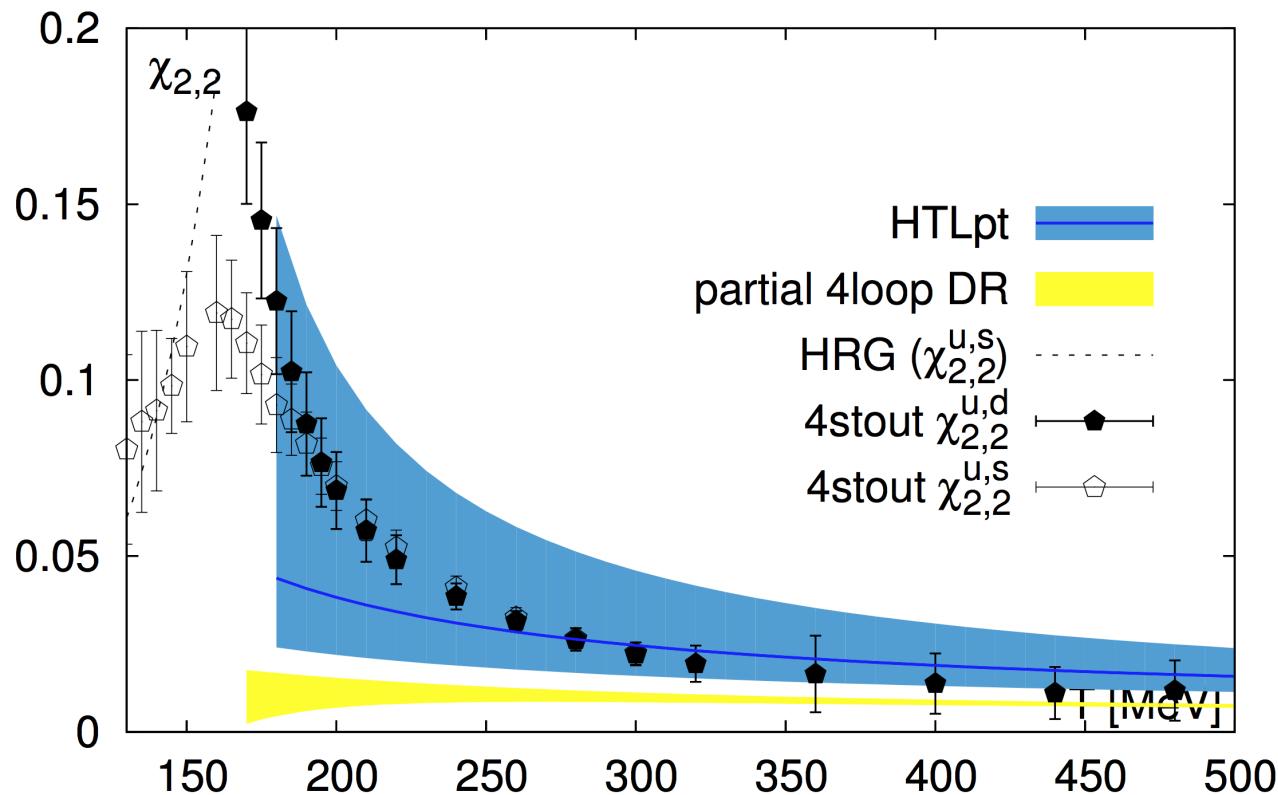
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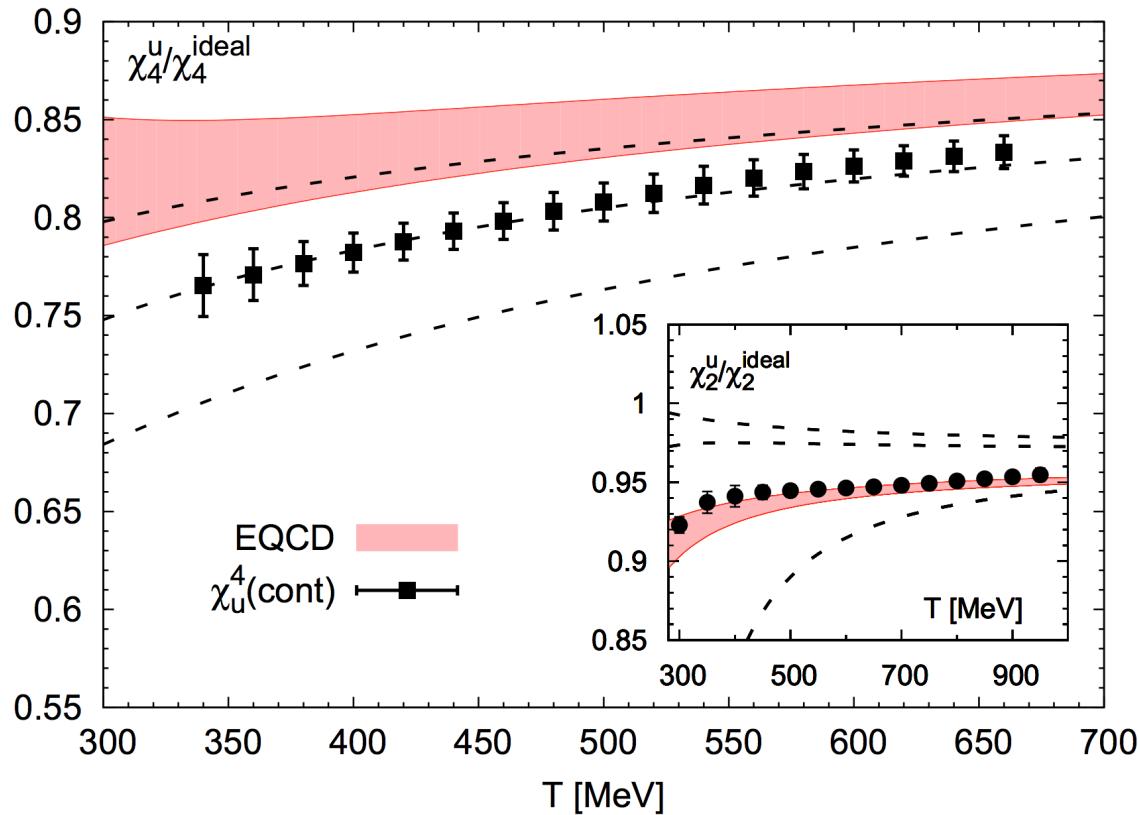
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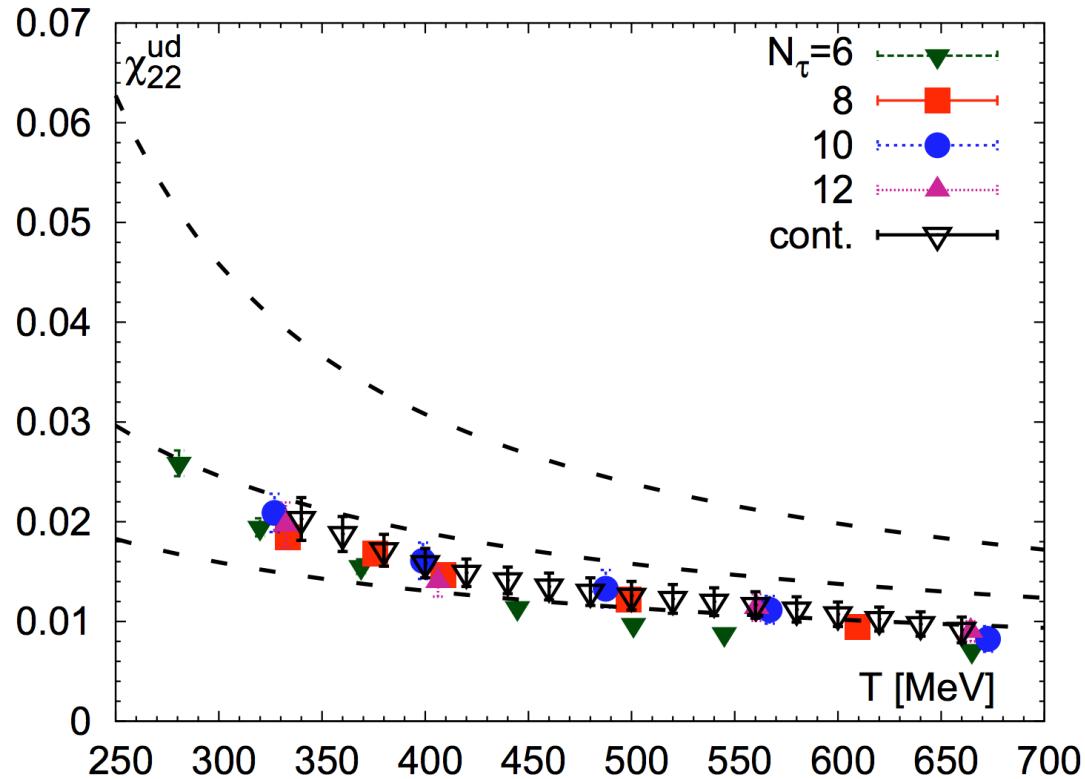
H.-T. Ding, Swagato Mukherjee, H. Ohno, P. Petreczky, H.-P. Schadler, 1507.06637



EQCD = dimensional reduction : S. Mogliacci, J.O. Andersen, MS, N. Su, and A. Vuorinen, 1307.8098

HTLpt = NNLO order : N. Haque, A. Bandyopadhyay, J.O. Andersen, M.G. Mustafa, MS, and N. Su, 1402.6907

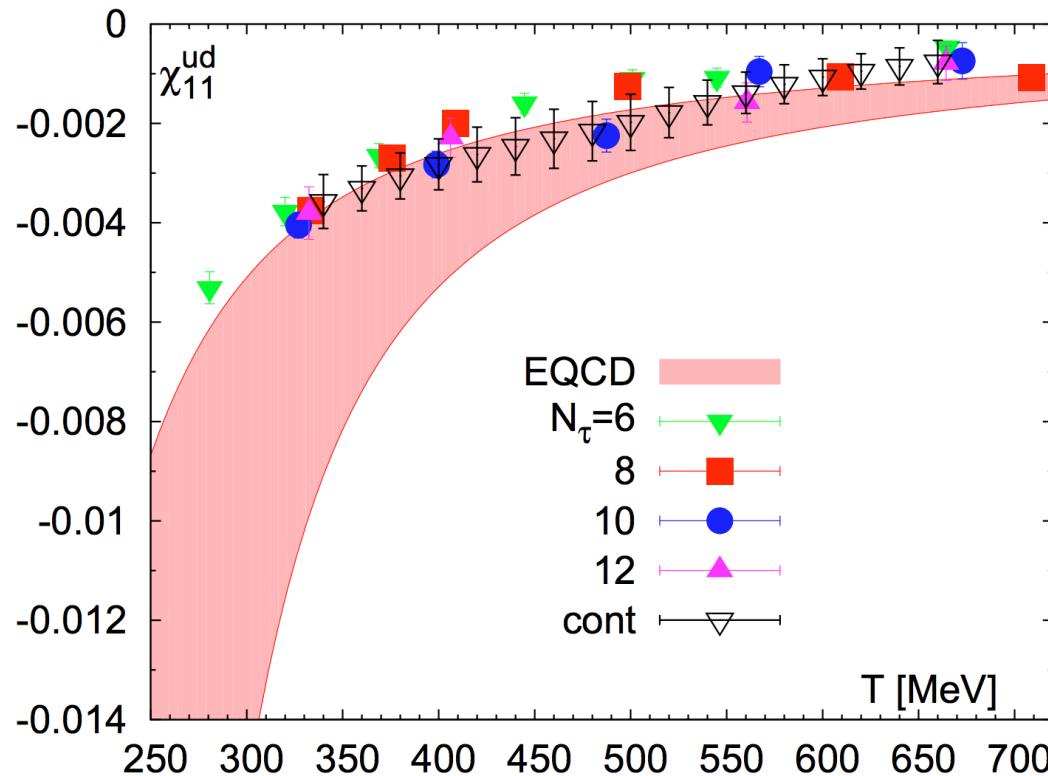
H.-T. Ding, Swagato Mukherjee, H. Ohno, P. Petreczky, H.-P. Schadler, 1507.06637



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H.-T. Ding, Swagato Mukherjee, H. Ohno, P. Petreczky, H.-P. Schadler, 1507.06637



EQCD = A. Hietanen and K. Rummukainen, 0802.3979

Conclusions and Outlook

- HTLpt reorganization → 3-loop QCD thermodynamic potential at finite temperature and chemical potential(s)
- Result is completely analytic and the agreement with the lattice data for a host of observables is quite nice considering that there are no fit parameters!
- For the susceptibilities it is also possible to use the resummed DR approach to compute them at four-loop order
- HTLpt is formulated in Minkowski space! It has already been applied to a host of non-equilibrium processes in the QGP (e.g. plasma instabilities, jet energy loss, quarkonium suppression, etc)
- These results give us some hope that application of HTLpt to the QGP is not misguided

Thanks to my collaborators



Najmul Haque



Sylvain Mogliacci



Nan Su



Aritra Bandyopadhyay



Jens Andersen



Munshi Mustafa

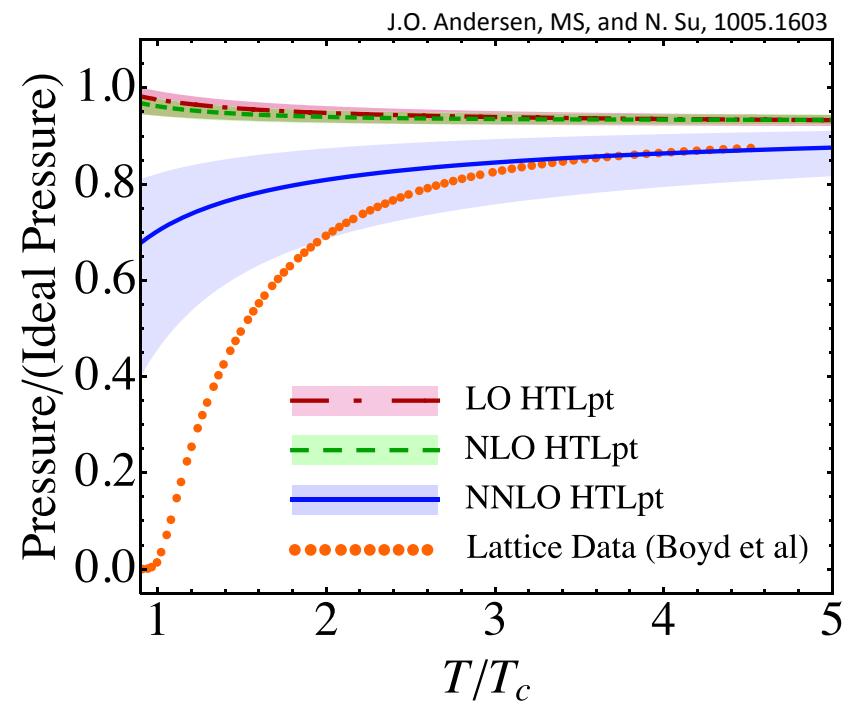
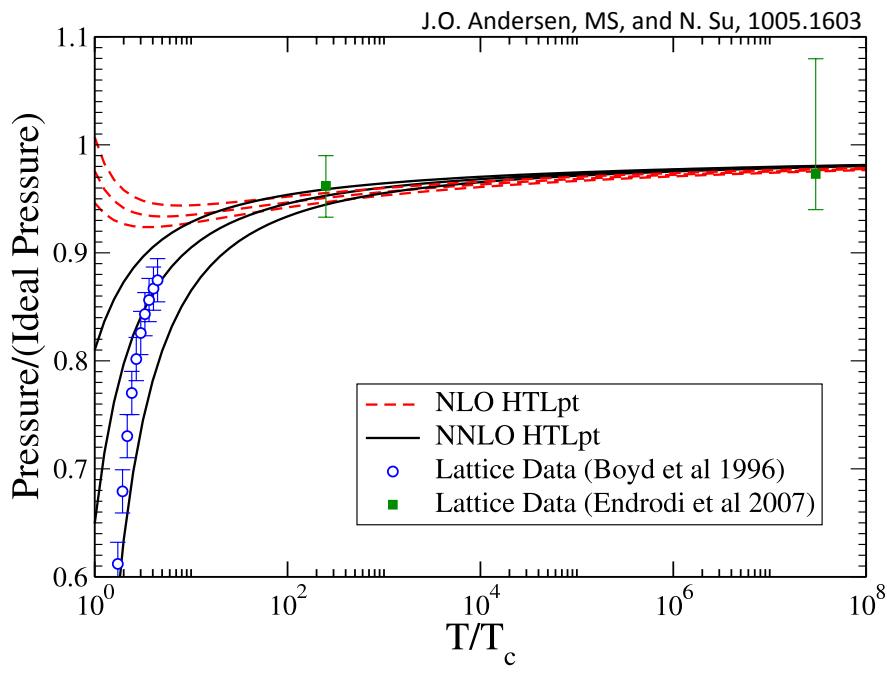


Aleksi Vuorinen

Backup Slides

HTLpt Convergence?

Pure Glue @ zero chemical potential



HTLpt Convergence?

